RT note: “I may need to cut or modify tis one but waiting for Gustavo”??

14.772
Spring, 2020

Where is Structure Needed, or Not: Imperfect Competition and Finance

(Lecture 7)

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Lecture 7: Where Structure is Needed, or Not: Imperfect Competition and Finance (3/31)


Related literatures and in other countries


And in Macro...


Optimal Contracting and Spatial Competition among Financial Service Providers


April 5, 2019
1. Theoretical Framework: Small and Medium Enterprises (SME) need credit/insurance from FSPs.
   - Contracting: FI, MH, LC, AdS
   - Market Structure: Spatial travel costs + logit errors.

2. Counterfactuals without specifying contracting friction.

3. Application to Townsend Thai Data
   - Model to data mapping.
   - Reducing spatial costs by 50% is equivalent to increasing consumption by 4.85%
Outline

Theoretical Framework

Frontier Identification

Application: Townsend Thai Data
Conceptual Framework, Model

- Change in competition space: utilities (not contracts), $u$. Consumption $c$, production $q$, effort $z$ and type $\theta$. Ex:

  \[ U(c(q), z | \theta) = \frac{c^{1-\sigma}}{1-\sigma} - \theta z^\sigma \]

- Risk-averse SMEs want to borrow, produce and consume. Production function with capital $k'$ given by probabilities:

  \[ P(q|k', \theta, z) \]

- FSPs ($b$): risk neutral offer credit, insurance s.t. financial constraints (MH, LC, AdS). Profits:

  \[ q - c(q) + (1 + r) \left[ (1 - \delta)k - k' \right] \]

- Structure on utility and production functions is not at all essential to our method.
Conceptual Framework, Model

- SMEs/FSPs spatially separated, travel cost from FSP $b$ to village $v$: $\psi t(x_v, x_b)$.
  - $t(x_v, x_b)$ measured through road network, actual travel times between points in map.

- Building Blocks: frontier + mkt structure

$$\Pi^b \equiv \underbrace{S(u_b)}_{\text{Surplus per unit } u_b} \underbrace{\mu(u_b, u_{-b}, u_0, x_b, x_{-b}, \{x_v\}_v)}_{\text{Demand at } u_b}$$

in village $v$ for bank $b$, competitors $-b$, outside option $u_0$.

- FOC (notation: $\partial_u f \equiv \frac{\partial f}{\partial u}$):

$$- \frac{\partial_u S(u_b^*)}{S(u_b^*)} = \frac{\partial_{u_b} \mu(u_b^*, u_{-b}, u_0, x_b, x_{-b}, \{x_v\}_v)}{\mu(u_b^*, u_{-b}, u_0, x_b, x_{-b}, \{x_v\}_v)}$$
Frontier: Contracting Building Block

- Given utilities, varying parametrically.
- Linear Programming given contracting frictions, chooses distribution
  \( \pi(c, z, q, k' | k, u) \). Let \( C = \{c, z, q, k'\} \):

\[
S(u | \theta) \equiv \max_{\pi(\cdot)} \sum_C \pi(C | k, u) \left[ q - c + (1 + r) \left[ (1 - \delta)k - k' \right] \right]
\]

- s.t.: promised utility constraint

\[
\sum_C \pi(C | k, u) U(c, z | \theta) = u
\]

- and financial frictions of the form

\[
\Gamma(\theta, u) \pi(C | k, u) \leq 0
\]

- Encompasses FI, MH, LC, AdS*
Figure: Frontier and Consumption Moments

- Observables moments + frontier allow inference of contracting obstacles and parameters.
Market Structure: Second building block

- Demand system: spatial + logit: \( u - \psi t(x_v, x_b) + \zeta \)

\[
\mu_b \equiv \sum_{v=1}^{V} N_v \left\{ \frac{e^{\sigma_L^{-1}[u_b - \psi t(x_b, x_v) - u_0]}}{1 + \sum_{b=1}^{B} e^{\sigma_L^{-1}[u_b - \psi t(x_b, x_v) - u_0]}} \right\}
\]

**Lemma.** Let the demand \( \mu \) be log-concave in \( u_b \), log-supermodular in \( (u_b, u_{-b}) \), bounded away from zero and satisfy \( \mu (u_b, u_{-b}, \{x_b\}) = \mu (u_b - a, u_{-b} - a, \{x_b\}) \) then the Nash equilibrium in utilities is unique and can be computed by an iteration of best responses.

- In some respects translate utility into price and we have standard methods, except we do not require quasi linear utility.
  - The problem we have is that with contracting and financial frictions, we don't know what production surplus looks like (the frontier) and must estimate it.
Equilibrium Existence and Computation

Figure: Nash Equilibrium: Monotonicity and Discounting

- Can identify surplus frontier (blue) with variations in supply side (red).
- Details to follow below.
Spatial Heterogeneity and the Role of Spatial Costs.
Local Competition and Information Structure.
Adverse Selection:
  - Theory.
  - Application: Local vs National Banks.
Taking to the Data

- Model has implications for $\mu_{v,b}$, SME $\{c, q, k\}$ data - as as in lead experimental example.

- $\mu_{v,b}$ data
  - Usual IO toolbox (logit regression)
  - Variation in competition allows for non-parametric identification of $S$, tracing out the frontier
  - no need to figure out obstacles: frontier is what it is for supply side counterfactuals:
    - Changes in spatial costs, road structure, number of FSPs (as in consumption-production example)

- $\{c, q, k\}$ data
  - Surplus Frontier + market structure $\rightarrow$ distribution of utilities and market shares $\rightarrow$ $\{c, q, k\}$ distribution by village and bank as as in lead experimental example.
  - Mapping depends on frictions + parameters.
  - Only need $c, q, k$ data to determine friction and utility levels.
  - With full structure: can also do counterfactuals changing contracting frictions (information structure changes as in example), and $\sigma_L$ (logit var)
Outline

Theoretical Framework

Frontier Identification

Application: Townsend Thai Data
Frontier Identification, Idea

- Each province $p$ (market) has different map configuration (# and position of FSPs).
- Notation

$$\Sigma(u) \equiv -\frac{\partial_{u_b} S(u_b)}{S(u_b)}, \quad \Upsilon(u, u_{-b}) \equiv \frac{\partial_{u_b} \mu(u_b, u_{-b}, u_0, x_b, x_{-b}, \{x_v\}_v)}{\mu(u_b, u_{-b}, u_0, x_b, x_{-b}, \{x_v\}_v)}$$

- In Equilibrium $\Sigma(u^*_b) = \Upsilon(u^*_b, u^*_{-b})$.
- Observe $\{\hat{\mu}_{v,b}^p\}$
  - Can compute marginal market share percentage gain, $\Upsilon(u^*_b, u^*_{-b})$, and through the FOC get a point on the y-axis of the frontier, $\hat{\Sigma}_p^b$.
  - Can invert market shares to recover utilities to get a point on x-axis of the frontier, $\hat{u}_b^p$.
- Not valid for all demand systems.
  - Need to be able to invert and to compute $\Upsilon$ (derivative based) from levels of market shares (which are observable).
Frontier Identification, Idea

Can identify surplus frontier (blue) with variations in supply side (red).

Figure: Regional Variation in Competition: provinces $p, \hat{p}$
Frontier Identification, Idea

- Without errors in the model, observe the black dots by varying the number of FSPs from 1-5 (increasing utility for more FSPs).
- The only difference between markets is number of FSPs in this case.
Frontier Identification, Method

- Include two errors in the model: measurement error/shocks to log market shares and model misspecification of FSPs. Two equations to identification:

\[
- \frac{\partial u_b S(u_b^*)}{S(u_b^*)} + \varsigma_b = \frac{\partial u_b \mu(u_b^*, u_{-b})}{\mu(u_b^*, u_{-b})} \tag{1}
\]

\[
\ln \left( \mu_{v,b}^p \right) - \ln \left( \mu_{0,v}^p \right) = \sigma_L^{-1} \left[ u_b^p - \psi t(x_b^p, x_v^p) - u_0 \right] + \vartheta_{b,v}^p \tag{2}
\]

- Eq. (1) is the FOC (with the shock), Eq. (2) comes from taking logs in mkt share equation and using the fact that sum of market share is 1.

- Utility scales \( \sigma_L \) and outside option \( u_0 \) not identified. Identify: \( \frac{\psi}{\sigma_L} \) and \( \sigma_L^{-1} \left[ u_b^p - u_0 \right] \), which are what is relevant for counterfactuals.
Frontier Identification: Method

- Estimate spatial cost $\tilde{\psi} \equiv \sigma^{-1}\psi$ from intra-market variation in market shares, utility offers are the 'fixed effects' - cleared out by within province means in the above equation. That is, computing intra-province variation of market shares:

$$
\ln \left( \mu_{v,b}^p \right) - \ln \left( \mu_{0,v}^p \right) - V^{-1} \sum_{v=1}^{V} \left[ \ln \left( \mu_{v,b}^p \right) - \ln \left( \mu_{0,v}^p \right) \right] =
$$

$$
- \tilde{\psi} \left\{ t(x_p^v, x_b) - V^{-1} \sum_{v=1}^{V} t(x_p^v, x_b) \right\} + V^{-1} \sum_{v=1}^{V} \vartheta_{p,b,v}
$$

- Estimate utilities through $\hat{u}_b^p = V^{-1} \sum_{v=1}^{V} \left[ \ln \left( \mu_{v,b}^p \right) - \ln \left( \mu_{0,v}^p \right) + \hat{\psi} t(x_p^v, x_b) \right]$

- Estimate $\Sigma$ from FSP FOC: $\hat{\Sigma}_b^p = 1 - \frac{\sum_{v=p} N_v^p (\hat{\mu}_{v,b}^p)^2}{\sum_{v=p} N_v^p \hat{\mu}_{v,b}^p} \left( = \hat{\Upsilon}_b^p \right)$

- RHS comes from derivative of logit market share $\partial_u \mu_{v,b} = \mu_{v,b} - \mu_{v,b}^2$.

- Dataset of $\{ \hat{u}_b^p, \hat{\Sigma}_b^p \}$ (at the village-province level).
Simulate data for 250 provinces, each with 50 villages (12,500 village-province combinations).

True spatial cost: $\psi = 1.5$.

FSPs located at uniformly distributed $x_L \in (0, .5)$ and $x_R = 1 - x_L$. (symmetric with respect to .5).

Randomly assign 2, 3 or 4 FSPs in $x_L$ (and an equal number to $x_R$).

True frontier (exogenous): $S(u) \equiv 1 - e^{2(u-1)}$. 
Frontier Identification, Results

- Estimating $\Sigma(u) = \frac{\partial S(u)/\partial u}{S}$ from $\{\hat{u}_b^P, \hat{\mu}_v^P\}$.

**Figure:** Identification of Frontier Through Market Share Data
Frontier Identification: The Role of Spatial Costs

- Use estimated frontier to compute counterfactuals.
- Here there is no structural model of utilities (or financial frictions - will put in our full structural estimation later).
- We scale utilities to the min/max observed in the data.
- Effect well estimated, increasing in competition.

**Table:** The Welfare Effect of Reducing Spatial Costs: from $\psi = 1.5$ to $\psi = .75$

<table>
<thead>
<tr>
<th>Baseline FSPs in ${x_L, x_R}$</th>
<th>True</th>
<th>Estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>.6166</td>
<td>.6168</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[ .6158, .6183]</td>
</tr>
<tr>
<td>3</td>
<td>.6724</td>
<td>.6728</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[.6719, .6737]</td>
</tr>
<tr>
<td>4</td>
<td>.7757</td>
<td>.7746</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[.7728, .7768]</td>
</tr>
</tbody>
</table>
Frontier Identification: Unobserved Heterogeneity of Villages

(a) \( \tilde{S}(u) = 1 - e^{2(u-1)} \)

(b) \( \tilde{S}(u) = 1 - e^{4(u-1)} \)

- Need a method that guarantees fitted function convex (theory constraint).
Frontier Identification: Unobserved Heterogeneity of Agents

- Observed controls: can compute different frontier.
- Here: two types of agents, observed by the FSP, but not by the researcher. Researcher only observes sum of market shares across two types.
Outline

Theoretical Framework

Frontier Identification

Application: Townsend Thai Data
Note: Chacheongsao province in terms of villages and Banks overall. Pink dots represent bank branches, black dots are villages and grey lines are the roads in 1999. Horizontal distance from extremes in the figure corresponds to $\approx 80$ miles.
Applied: Townsend Thai Data

- 1999 Data from Townsend Thai Data Monthly Survey.
- Data for 531 households, across 16 villages, 4 Provinces.
- Buriram and Sisaket: located in the north-east region, which is relatively poor and semi-arid, with, respectively, 1 and 2 FSPs in a 30min radius.
- Chacheongsao and Lopburi: located near Bangkok and, in part, urban, with, respectively, 15 and 4 FSPs in a 30min radius.
- Potential entry: locations that had a bank from 1970-2011.
## Table: Summary Statistics

<table>
<thead>
<tr>
<th>Consumption expenditure, $c$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>58,311</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>48,951</td>
</tr>
<tr>
<td>Median</td>
<td>43,895</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Production, $q$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>100,820</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>290,997</td>
</tr>
<tr>
<td>Median</td>
<td>42,013</td>
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</table>

<table>
<thead>
<tr>
<th>Business Assets, $k$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>76,065</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>401,008</td>
</tr>
<tr>
<td>Median</td>
<td>10,959</td>
</tr>
</tbody>
</table>

Average exchange rate in 1999-2000 was 1 USD = 39 Baht.
Estimation: Townsend Thai Data

- Use full structure of the model - not only contracting or market structure blocks (as done before).
- From SME data
  - \{c, q, k\} distribution by village and bank, bank market shares and profits implied by equilibrium.
  - Extend Karaivanov and Townsend (2014) to include market shares and different utilities in \{c, q, k\} distribution.
    - Model does not perfectly predict \{c, q, k\} in the data due to grids/simplifications: measurement error with variance \(\gamma_{ME}\) from model to the data.
    - Likelihood by village based on bank market shares and equilibrium utilities in each.

- From FSP Profits: extra FSP should have negative profit in potential locations
  - Bresnahan and Reiss (1991) entry model.
  - Location idiosyncratic profit shock \(\iota \sim \mathcal{N}(-c_E, s)\) (cost of entry).
Model implies: \[ \{ \pi_v^p(c, q, z| k, u_{b \in p}^*) \}_{v}^p \]

Data on households \( \{ \hat{y}_j \}_{j=1}^{H} \), where \( j = 1, ..., H \).

We use: \( y_j = (c_j, q_j, k_j) \)

Deal with actual measurement error/fitting the data into discrete grids

\[ \mathcal{N}(0, \gamma_{ME} \cdot \chi^2(X)) \]

where \( \chi^2(X) \) denotes the range of the grid \( X = C, K, Q \).

Structural parameters \( \zeta \).
Likelihood 2

- We can write the density for \((c, q)\) as

\[
g_v(c, q|k, \zeta) = \sum_u m_v^u(k) \sum_z \pi(c, q, z|k, u) + \left[1 - \sum_u m_v^u(k)\right] \sum_z \pi^{aut}(c, q, z|k)
\]

- Where \(m_v^u(k)\) is the share of agents in village \(v\), capital \(k\) that are offered utility \(u\) by a FSP

\[
m_v^u(k) \equiv \sum_{b \in B} \sum_u \mathbb{1}_{u = u^*(b)} \mu_v^b(u, k)
\]

- Multiply by observed distribution of capital

\[
f_v(c, q, k|\zeta) = g_v(c, q|k, \zeta) h_v^k(k)
\]

- Note: without capital data (or other observed heterogeneity) - can parametrize this function as Karaivanov and Townsend (2014). Can be applied to compute equilibrium utilities without a model for market structure.
Define $\#Y \equiv C \times Q \times K$, $l = 1, \ldots, L$ represents the different elements of $Y$.

For household data in a given village

$$F_v (\hat{y}_j; \zeta, \gamma_{ME}) \equiv \sum_{r=1}^{\#Y} f_v(c, q, k|\zeta) \prod_{l=1}^{L} \Phi \left( \hat{y}^l_j \mid y^l_r, \zeta^l(\gamma_{ME}) \right)$$

For FSP location data

$$\Pi^F (B_{m^p}|m^p, \{x^p_b\}_{b \notin m^p}) \equiv \Pi^E (B_{m^p}|m^p, \{x^p_b\}_{b \notin m^p}) + \tau_{m^p}, \quad \tau_{m^p} \sim \mathcal{N}(-c_E, s)$$

Brenahan and Reiss (1991): at $B$ banks profits are positive, at $B + 1$ they should be negative.
Likelihood: Combining Supply and Demand

- $S_p$: Supply Data in province $p$: FSPs locations/potential entry data (Bresnahan and Reiss, 1991)
- $D_p$: $(c, q, k)$ data and village for HHs.
- To combine, can take position of FSPs as given to compute demand, sum with supply:

$$L(\zeta|S_p \in P, \{D_p\}_p \in P) = \sum_{p \in P} \ln [\mathbb{P}(S_p|\zeta)] + \sum_{p \in P} \ln [\mathbb{P}(D_p|\zeta, S_p)]$$  

(3)

- Overall, must estimate $\{\psi, \sigma_L, \gamma_{ME}, s, c_E\}$.
  - Calibrate $\{\sigma, \theta, \varphi\}$.
  - Frontier only needs to be computed once (building block).
  - Computed through Gurobi linear solver.
In our supply/demand separation, can write:

$$\max_{\psi, \sigma_L} \left( \max_{c_E, s} \sum_{p \in P} \ln \left[ P \left( \mathcal{S}_p | \{ \psi, \sigma_L, c_E, s \} \right) \right] + \max_{\gamma_{ME}} \sum_{p \in P} \ln \left[ P \left( \mathcal{D}_p | \{ \psi, \sigma_L, \gamma_{ME} \}, \mathcal{S}_p \right) \right] \right)$$

Given $\psi, \sigma_L$

- Compute Equilibrium: iteration of best responses.
- Can determine $\{\gamma_{ME}, s, c_E\}$ optimally through FOC of likelihood.

Optimize $\psi, \sigma_L$ through grid search + global optimization toolbox.

Identification of $\psi, \sigma_L$

- Monte Carlo experiments show we can identify parameters numerically.
- Heterogeneous ($\psi$) vs Homogeneous effects ($\sigma_L$) across villages.
Counterfactuals: Townsend Thai Data

- Baseline: parameter estimates.
- Consumption vs Welfare, as in the example in the beginning.
- Policy: focus on inducing FSPs to compete, rather than increasing players.

Table: % Change from baseline

<table>
<thead>
<tr>
<th></th>
<th>.5ψ</th>
<th>.75ψ</th>
<th>.5σ_L</th>
<th>.75σ_L</th>
<th>Bank Entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welfare (Cons. Equiv.)</td>
<td>4.85</td>
<td>1.37</td>
<td>9.21</td>
<td>3.70</td>
<td>2.20</td>
</tr>
<tr>
<td>Average Consumption</td>
<td>-1.97</td>
<td>-0.70</td>
<td>0.89</td>
<td>1.05</td>
<td>-2.72</td>
</tr>
<tr>
<td>Std. in Consumption</td>
<td>-8.33</td>
<td>-2.78</td>
<td>5.46</td>
<td>1.43</td>
<td>-12.33</td>
</tr>
</tbody>
</table>
Conclusion

- Interpreting reduced form evidence challenging with contracting + competition.

- Two building blocks: frontier + market structure.

- Flexible contracting specification, can be taken to the data $\rightarrow$ empirical toolkit.

- Depending on the effects or counterfactuals of interest, do not need to specify structure on both ingredients.
Eq. (4) is the *Incentive Compatibility Constraint*: when effort is not observed, it is optimal for the agent to execute the effort recommended by the FSP.

Eq. (5) if the FSP can recover \((1 - \rho)\) of the output, the utility offered are such that the household has incentives to repay if it can keep the remaining \(\rho\) share of the output.

\[
\sum_{c,q,k'} \pi(c, q, \bar{z}, k') \mathbb{U}(c, \bar{z}|\theta) \geq \sum_{c,q,k'} \pi(c, q, \bar{z}, k') \mathbb{U}(c, \hat{z}|\theta) \frac{P(q|k', \theta, \hat{z})}{P(q|k', \theta, \bar{z})} \ \forall \bar{z}, \hat{z} \in Z, \ \forall \theta \quad (4)
\]

\[
\mathbb{U}(\rho \bar{q}, \bar{z}|\theta) \leq \sum_{c} \pi(c, \bar{q}, \bar{z}, \bar{k'}) \mathbb{U}(c, \bar{z}|\theta), \ \forall \bar{q}, \bar{z}, \bar{k'} \in Q \times Z \times K, \ \forall \theta \quad (5)
\]
## Frontier: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Constraint</th>
<th>Role</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>1.5</td>
<td>Risk Aversion</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>2</td>
<td>Disutility of Effort</td>
</tr>
<tr>
<td>$\theta$</td>
<td>1</td>
<td>Effort Multiplier</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.25</td>
<td>Share of Non-Recoverable Assets</td>
</tr>
</tbody>
</table>

**Table: Parameter Values, Grids and Constraints for Frontier Construction**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Grid</th>
<th># Points</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>[0.04, 1.75]</td>
<td>5</td>
<td>$10^{th}$, $30^{th}$, ..., $90^{th}$ p-tile in data</td>
</tr>
<tr>
<td>$K$</td>
<td>[0, 1]</td>
<td>5</td>
<td>$10^{th}$, $30^{th}$, ..., $90^{th}$ p-tile in data</td>
</tr>
<tr>
<td>$Z$</td>
<td>[0,1]</td>
<td>3</td>
<td>uniform</td>
</tr>
<tr>
<td>$C$</td>
<td>[0.001, 1.75]</td>
<td>64</td>
<td>uniform</td>
</tr>
<tr>
<td>$W$</td>
<td>[$w_{min}$, $w_{max}$]</td>
<td>150</td>
<td>uniform</td>
</tr>
</tbody>
</table>
### Table: Baseline Parameters used for Comparative Statics Exercises

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>1</td>
<td>Spatial Cost</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>.33</td>
<td>Logit Variance</td>
</tr>
<tr>
<td>$V$</td>
<td>50</td>
<td>Number of Villages</td>
</tr>
<tr>
<td>$b_L$</td>
<td>1</td>
<td>Number of FSPs in $x = 0$</td>
</tr>
<tr>
<td>$b_R$</td>
<td>2</td>
<td>Number of FSPs in $x = 1$</td>
</tr>
</tbody>
</table>
## Parameters AdS

**Table:** Baseline Parameters used for AdS Comparative Statics Exercises

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_L$</td>
<td>1</td>
<td>Low Type</td>
</tr>
<tr>
<td>$\theta_H$</td>
<td>2</td>
<td>High Type</td>
</tr>
<tr>
<td>$f_L$</td>
<td>.5</td>
<td>Share of Low Type in each Village</td>
</tr>
<tr>
<td>$f_H$</td>
<td>.5</td>
<td>Share of High Type in each Village</td>
</tr>
<tr>
<td>$\psi$</td>
<td>1</td>
<td>Spatial Cost</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>.1</td>
<td>Logit Variance</td>
</tr>
<tr>
<td>$V$</td>
<td>50</td>
<td>Number of Villages</td>
</tr>
<tr>
<td>$b_L$</td>
<td>1</td>
<td>Number of FSPs in $x = 0$</td>
</tr>
<tr>
<td>$b_R$</td>
<td>1</td>
<td>Number of FSPs in $x = 1$</td>
</tr>
</tbody>
</table>
Numerical Identification, market structure $\psi, \sigma_L$

Note: Likelihood of household level data. Red line is the true value, dotted blue line is log-likelihood. Data simulated for four provinces, with 10 villages in each road, each of them with $N = 75$ households.
Numerical Identification, market structure $\psi, \sigma_L$

Figure: Log-Likelihood of Household Level Data as a Function of Spatial Cost ($\psi$) and Logit Var ($\sigma_L$) for Simulated data

Note: Likelihood of household level data. Red line is the true value, dotted blue line is log-likelihood. Data simulated for four provinces, with 10 villages in each road, each of them with $N = 75$ households.
Numerical Identification, frontier parameters $\sigma$, $\varphi$

(a) Risk Aversion ($\sigma$)  
(b) Disutility of Effort ($\varphi$)

Figure: Log-Likelihood of Household Level Data as a Function of Risk Aversion ($\sigma$) and Disutility of Effort ($\varphi$) for Simulated data
Theoretical Identification, Market structure $\sigma, \theta$

- $\psi_L$: Heterogeneous effects for villages located in $x \in \{0, .5, 1\}$
- $\sigma_L$: homogeneous effects

Figure: Welfare of villages located in $x \in \{0, .5, 1\}$ as a function of $\psi$
Optimal \( \{\gamma_{ME}, s, c_E\} \)

\[
\sum_p \sum_{v^p} \sum_{j^p} \mathbb{1}_{\varphi_j^p = v^p} \frac{\sum_{r=1}^{\#Y} f_v(c, q, k | \zeta) \exp \left\{ \sum_{l=1}^{L} - \frac{(\hat{y}_l^r - y_l^r)^2}{2 \hat{\lambda}_l^2 \hat{\gamma}_{ME}} \right\} \left[ \sum_{l=1}^{L} \frac{(\hat{y}_l^r - y_l^r)^2}{\hat{\lambda}_l^2 \hat{\gamma}_{ME}} \right]}{\sum_{r=1}^{\#Y} f_v(c, q, k | \zeta) \exp \left\{ \sum_{l=1}^{L} - \frac{(\hat{y}_l^r - y_l^r)^2}{2 \hat{\lambda}_l^2 \hat{\gamma}_{ME}} \right\}} = 1 + L \quad (6)
\]

\[
\sum_{m^p} \frac{\phi \left[ \frac{\Pi^E(B_{mp} + 1 | .)}{\hat{s}} \right] - \phi \left[ \frac{\Pi^E(B_{mp} | .)}{\hat{s}} \right]}{\Phi \left[ \frac{\Pi^E(B_{mp} | .)}{\hat{s}} \right] - \Phi \left[ \frac{\Pi^E(B_{mp} + 1 | .)}{\hat{s}} \right]} = 0 \quad (7)
\]

\[
\sum_{m^p} \frac{\phi \left[ \frac{\Pi^E(B_{mp} + 1 | .)}{\hat{s}} \right] \Pi^E(B_{mp} + 1 | .) - \phi \left[ \frac{\Pi^E(B_{mp} | .)}{\hat{s}} \right] \Pi^E(B_{mp} | .)}{\Phi \left[ \frac{\Pi^E(B_{mp} | .)}{\hat{s}} \right] - \Phi \left[ \frac{\Pi^E(B_{mp} + 1 | .)}{\hat{s}} \right]} = 0 \quad (8)
\]
### Table: Parameter Estimates

<table>
<thead>
<tr>
<th>Estimate</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\gamma}_{ME}$</td>
<td>.21 Measurement Error</td>
</tr>
<tr>
<td>(0.0139)</td>
<td></td>
</tr>
<tr>
<td>$\hat{\psi}$</td>
<td>.55 Spatial Cost</td>
</tr>
<tr>
<td>(0.0175)</td>
<td></td>
</tr>
<tr>
<td>$\hat{\sigma}_L$</td>
<td>.083 Logit. Var</td>
</tr>
<tr>
<td>(0.005)</td>
<td></td>
</tr>
<tr>
<td>$\hat{c}_E$</td>
<td>1.57 Cost of Entry</td>
</tr>
<tr>
<td>(0.0260)</td>
<td></td>
</tr>
<tr>
<td>$\hat{s}$</td>
<td>0.03 Variance of Location Specific Profit Shock</td>
</tr>
<tr>
<td>(0.0001)</td>
<td></td>
</tr>
</tbody>
</table>
Distance to Nash

Let $G$ be a set of strategies by both players and $P_1(G)$ and $P_2(G)$ their payoffs. Let $G_1$ be strategies of player 2 - that is, all that is necessary for 1 to compute its best response (and equivalently $G_2$).

$$d(G, G_1) = \max(P_1(G_1) - P_1(G), 0)$$
$$d(G, G_2) = \max(P_2(G_2) - P_2(G), 0)$$

In the first step of procedure we compute $P_1(G)$ and $P_2(G)$ for a trial strategy set of $G$. Then, in the second stage we solve

$$\max_{G_1} d(G, G_1) \text{ subject to } P_1(G) > 0, \forall G_1.$$ 

and the same for $d(G, G_2)$. Let $d(G, G_1)$ denote the solution of the problem above. We compute distance to Nash as

$$d(G, G_1, G_2) = d(G, G_1) + d(G, G_2)$$

And in the final stage we solve

$$\min_G d(G), \forall \{G, G_1, G_2\}.$$
Complementarity of $\psi$ and $\sigma_L$

- Welfare in equilibrium changing the logit variance, denoted by $\sigma_L$, and spatial costs, denoted by $\psi$. One FSP at $x = 0$ and one at $x = 1$, that is $b_L = b_R = 1$.

Figure: Average Welfare varying spatial costs and logit variance
Lecture 7: Where Structure is Needed, or Not: Imperfect Competition and Finance (3/31)


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