Economies

• Two parts
  • Actual example we will use in the course
  • Language of General Equilibrium
    • Commodity Space
    • PET
      • Preference
      • Endowment
      • Technology
Medieval Village Economy
A map of landholdings in the village of Elford, Staffordshire, England, 1719, before enclosures, reveals a striking geometric pattern of long narrow strips. The holdings of one Mr. T. Darlaston are shaded in black, showing how fragmented was his land, with many separated holdings throughout this three-square-mile village.

Endowments
Fig. 1. Map of Elford Staffordshire, 1719. Reprinted, by permission, from George C. Homans, *English Villagers of the Thirteenth Century*, 88
Fig. 2. Map of the Mill Field of Laxton. Reprinted, by permission, from C. S. and C. S. Orwin, *The Open Fields*. 
Risk, States of the World

• Harvest yields, output per unit land, from the manors of the Bishop of Winchester, England, 1209-1350 reveal high variability. Coefficients of variation, the sample standard deviation divided by the mean.

<table>
<thead>
<tr>
<th>TABLE I</th>
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<tr>
<td>Average Coefficient of Variation for Three Crops on the Winchester Demesnes, 1335-49</td>
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<tr>
<td>Number of Demesnes</td>
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<td>Average Coefficient of Variation (Net of Seed)</td>
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<td>Standard Deviation of the Average</td>
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<td>Standard Error of the Average</td>
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Source: Donald McCloskey, "English Open Fields as Behavior Towards Risk," 134.
With a coefficient of variation of .35, available output would fall below half of mean value every twelve years or so. That is, it would fall in the critical lower tail, below $D$, of the distribution marked "scattered" in figure 3. English villages suffered from famine at roughly this frequency in this early period.
Correlations of harvest yields taken pairwise across villages reveal that correlations fall with distance, roughly ten points per mile (McCloskey 1976). average correlations across near villages (less than ten miles apart) and far villages

Location

Winchester estates were spread out over various counties in southern England, often separated by considerable distances. See the map taken from Titow (1972)
Dynamics, Time

• The same estate accounts show that grain still in storage at the time of harvest, termed carryover, is highly jagged and often zero. Surprisingly, there was little or no storage in this sense.

• The estate accounts also record no evidence whatsoever for borrowing and lending between lord and villagers.
Farming Technology: Part of PET

Farming is relatively simple in Yang Pieng:

- use of herbicides and pesticides is rare
- fertilizer is occasionally mentioned
- Local tractors (motorized tillers) are rented within the village
- most mention of "hired" labor is exchange labor
- inputs and outputs can be bought in a district town about twelve km away
- rice is valued at 30 baht per tang
Livestock can be a source of income for some farmers in addition to farming. Pigs are bought at prices ranging from 300 to 600 baht and sold at prices from 900 to 1500 baht, with higher prices for older, more mature animals.

The members of a typical household are endowed with time used for leisure and for various production activities. Some work in the local village economy or the local district economy, at wages of 30 to 35 baht per day.

Employment and income-generating activities associated with northern Thai forests: gathering bamboo and mushrooms; illegal logging can be a source of income, although this is increasingly difficult in the face of increased enforcement by forestry officials.
Local and Regional Variation
Variation across Villages

• From small (village) to regional (collection of villages) to large (national)
• An intriguing stylized fact from the field research is substantial variation over villages
• This is no more apparent than in amphoe Maajam where the three villages of Mae Wak, Sop Wak, and Maanajohn lie only a few miles from one another, all within walking distance of one another, but yet differ substantially on key dimensions.
• Specifically, Mae Wak is loaded with institutions; Sop Wak, down the road, has difficulty with the same funds; and Maanajohn has few institutions of any kind.
Deterioration with Growth

- A final village is of some interest. Ba Pai in amphoe Lee lies along both sides of a major highway to Bangkok.
- It is much more involved in the commercial economy of Thailand than any other village in this field research.
- Its institutions and internal markets are also different.
Townsend Thai Project
U.S. Regional and Local Variation: Geography
China Shock
New York Financial Markets

Chart 1
Settlement Fails in U.S. Treasury Securities


Note: The chart plots average daily delivery fails of the primary dealers for the week ending July 4, 1990, through the week ending December 29, 2004.

Economics: Language of General Equilibrium
PET Economies: Introduction and General Setup

Let $L > 0$ be the (finite) number of commodities in an economy, $I > 0$ the number of consumers and $J \geq 0$ the number of firms.

1. Each consumer $i = 1, 2, ..., I$ has a consumption set $X_i \subseteq \mathbb{R}_+^L$ and preferences over bundles on $X_i$

2. Each firm $j = 1, 2, ..., J$ has a technology, characterized by a production set $Y_j \subseteq \mathbb{R}^L$

3. The economy resources are given by a vector of aggregate endowments $\bar{\omega} = (\bar{\omega}_1, \bar{\omega}_2, ..., \bar{\omega}_L) \in \mathbb{R}^L$
Extension of Commodity Space

- Time - Dynamics
- Geography - Location
- Uncertainty - States of the World
Commodity Space Interpretation: Time

- Preferences
  - Utility: \( \sum_{t=1}^{T} \beta^t U(c_t) \)
  - Possibly infinite, \( T = \infty \): \( \sum_{t=1}^{\infty} \beta^t U(c_t) \)

See future lecture on techniques for infinite horizon
The Transportation Problem: An LP Formulation

Suppose a company has \( m \) warehouses and \( n \) retail outlets. A single product is to be shipped from the warehouses to the outlets. Each warehouse has a given level of supply, and each outlet has a given level of demand. We are also given the transportation costs between every pair of warehouse and outlet, and these costs are assumed to be linear. More explicitly, the assumptions are:

- The total supply of the product from warehouse \( i \) is \( a_i \), where \( i = 1, 2, \ldots, m \).
- The total demand for the product at outlet \( j \) is \( b_j \), where \( j = 1, 2, \ldots, n \).
- The cost of sending one unit of the product from warehouse \( i \) to outlet \( j \) is equal to \( c_{ij} \), where \( i = 1, 2, \ldots, m \) and \( j = 1, 2, \ldots, n \). The total cost of a shipment is linear in the size of the shipment.

The problem of interest is to determine an optimal transportation scheme between the warehouses and the outlets, subject to the specified supply and demand constraints.

A transportation scheme is a complete specification of how many units of the product should be shipped from each warehouse to each outlet. Therefore, the decision variables are:

\[
x_{ij} = \text{the size of the shipment from warehouse } i \text{ to outlet } j, \text{ where } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n.
\]

This is a set of \( m \times n \) variables.
The Objective Function

Consider the shipment from warehouse $i$ to outlet $j$. For any $i$ and any $j$, the transportation cost per unit is $c_{ij}$; and the size of the shipment is $x_{ij}$. Since we assume that the cost function is linear, the total cost of this shipment is given by $c_{ij}x_{ij}$. Summing over all $i$ and all $j$ now yields the overall transportation cost for all warehouse-outlet combinations. That is, our objective function is:

$$\text{Minimize } \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}.$$  

LP Formulation

In summary, we have arrived at the following formulation:

$$\begin{align*}
\text{Minimize} & \quad \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \\
\text{Subject to:} & \quad \sum_{j=1}^{n} x_{ij} \leq a_i \quad \text{for } i = 1, 2, \ldots, m \\
& \quad \sum_{i=1}^{m} x_{ij} \geq b_j \quad \text{for } j = 1, 2, \ldots, n \\
& \quad x_{ij} \geq 0 \quad \text{for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n.
\end{align*}$$

This is a linear program with $m \times n$ decision variables, $m + n$ functional constraints, and $m \times n$ nonnegativity constraints.
States of the World: Uncertainty

• Debreu (1959), 7.3, Fig 1

- Expected utility: $\sum_{t=0}^{T} \text{prob}(s^t|s_0) U[c(s^t)]$
- At date $t$ given state $s_t$ and history $s_0, s_1, \ldots, s_{t-1}$, index consumption by history $c(s^t)$
- Securities paying $R(s^t), \forall s^t$
- All planned at $t = 0$, but could allow spot markets and re-trading
  - Contrast, incomplete markets, then $\Rightarrow$ limited securities at $t = 0$ matter, shutting down exogenously the space of trades.
  - Debt: pay constant amount but allow bankruptcy

See future lecture on village risk sharing
Households, How Many?

• Overlapping generations
  – Classic example
  – Time and number = infinity
AN EXACT CONSUMPTION-LOAN MODEL OF INTEREST WITH OR WITHOUT THE SOCIAL CONTRIVANCE OF MONEY

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My first published paper has come of age, and at a time when the subjects it dealt with have come back into fashion. It developed the equilibrium conditions for a rational consumer's lifetime consumption-saving pattern, a problem more recently given by Harrod the useful name of "hump saving" but which Landry, Böhm-Bawerk, Fisher, and others had touched on long before my time. It dealt only with a single individual and did not discuss the mutual determination by all individuals of the market interest rates which each man had to accept parametrically as given to him.

Now I should like to give a complete general equilibrium solution to the determination of the time-shape of interest rates. This sounds easy, but actually it is very hard, so hard that I shall have to make drastic simplifications in order to arrive at exact results. For while Böhm and Fisher have given us the essential insights into the pure theory of interest, neither they nor other writers seem to have grappled with the following tough problem: in order to define an equilibrium path of interest in a perfect capital market endowed with perfect certainty, you have to determine all interest rates between now and the end of time; every finite time period points beyond itself!

Some interesting mathematical boundary problems, a little like those in the modern theories of dynamic programming, result from this analysis. And the way is paved for a rigorous attack on a simple model involving money as a store of value and a medium of exchange. My essay concludes with some provocative

* Research aid from the Ford Foundation is gratefully acknowledged.


2 As an undergraduate student of Paul Douglas at Chicago, I was struck by the fact that we might, from the marginal utility schedule of consumptions, deduce saving behavior exactly in the same way that we might deduce gambling behavior. Realizing that, watching the consumer's gambling responses to varying odds, we could deduce his numerical marginal utilities, it occurred to me that, by watching the consumer's saving responses to varying interest rates, we might similarly measure his marginal utilities, and thus the paper was born. (I knew and pointed out, p. 155, n. 2; p. 160, that such a cardinal measurement of utility hinged on a certain refutable "independence" hypothesis.)
Population of US by age and sex

See future lecture on inefficiency
Monetary Economics

- Turnpike Model of Exchange
  - Valued outside fiat money
    - Townsend

![Figure 1: The Turnpike Model](image)

![Table V-1. Who Meets Whom When](image)

Townsend and Wallace (1987)
Circulating private debt, paper IOUs which passed from hand to hand, emerged also, though this took some time. As was noted above, Italians settled in places like Bruges as representatives of home trading companies or banks. This allowed parties in distinct cities to issue bills of exchange. For example, a partner in Bruges would borrow money from a merchant in Bruges and instruct his partner in Florence to pay back the loan in Florence at a specified future date, either to the lender or to the latter’s trading partner. In effect, this was an exchange contract described earlier except that the second partner, not the issuer, was obliged to pay. Active markets in bills, with many merchant-suppliers and merchant-demanders, emerged in Bruges by 1420, and their dominance continued in the fourteenth and fifteenth centuries. Later Antwerp dominated the bill market. Finally, as noted, bills began to circulate, in Bruges by 1577, in London by 1600, in France by 1604, and in Antwerp in 1610, according to Usher (1943). Earlier predecessors uncovered by Usher include two unfolded unsealed notes payable to the bearer, dated 1399 and 1400. English fair bonds, described below, may have been negotiable by the beginning of the fourteenth century, though Usher disputes this.