Monetary Economics

• Turnpike Model of Exchange
  – Valued outside fiat money
  • Townsend

![Turnpike Model Diagram]

Table V-1. Who Meets Whom When

<table>
<thead>
<tr>
<th>Date</th>
<th>Location</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1,2)</td>
<td>3,4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(1,3)</td>
<td>2,4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(1,2)</td>
<td>3,4</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(1,3)</td>
<td>2,4</td>
<td></td>
</tr>
</tbody>
</table>

Townsend and Wallace (1987)
Circulating private debt and multiple media of exchange

Townsend and Wallace (1987) provide an instructive example environment. There are four agents, four periods, and two locations. Agents have endowments of a consumption good that varies over time, but again, as earlier, one can generalize and imagine these are other objects. One can trace out chains of named debt from the issuer at the issue date passing through third parties to the issuer at the redemption date (here no reneging is allowed). This circulating private debt is the medium of exchange in contemporaneous transactions, supporting trade in other short-term non-circulating securities and the consumption commodity. One can take the consumption good as the numeraire, but again, as earlier, this could be generalized.
The point of the model is that there is a coordination problem. There are a large number of potential equilibria, each of which achieves the same target Pareto optimal, complete markets equilibrium real allocation. But these equilibria vary in who is issuing the debt initially, hence what objects are circulating, that is, what objects are providing the payments device. In the key example, to be specific, agents 1 and 2 are matched in location one and agents 3 and 4 are matched at location two. Agents 1 and 4 stay put at their respective locations, but agents 2 and 3 keep switching back and forth from one period to the next. There are many equilibria that achieve the Pareto optimal target, either all the debts that are allowed to circulate could be issued by initial parties in one of the two locations, or by the parties in the other, second location. Or, they could be issued in various particular convex combinations. But by assumption, in the informationally decentralized model environment, there is no way for traders in one location to know what is going on in the other. Too much or too little debt as liquidity could be issued.
This can cause problems later in subsequent markets. Some circulating debts would be “over-issued” resulting in a precipitous drop in their prices later on in the trading cycle. It seems failure to achieve coordination links up with observed chaotic conditions. Bills of exchange were traded in London money markets, and these crashed, leading to arguments for the creation of a central bank.

DLT keeping track and verifying initial issues of long-term debt in exchange for consumption or other objects, if public, would achieve in the example environment, the necessary coordination. Interestingly, there are no liquidity premia associated with the circulating private debt, to pick up from the data, yet the coordination problem remains. An understanding of the environment and data tracking will be needed.
Lessons from monetary theory in Walrasian, competitive markets

A lesson from monetary theory is that fiat money can have value even if intrinsically worthless. The same arguments can apply to some cryptocurrencies though we need a derivation, as below, to get to that conclusion.
Bitcoin bubble grows and grows
This time, Wall St is innocent.
To the long list of asset bubbles – from tulips to the South Sea Company, from dotcom stocks to US housing – economic historians may soon be adding a virtual “currency” called Bitcoin. But while it is bankers who are most often blamed for blowing up bubbles, the rise and rise in the Bitcoin price has taken place without any such intervention.
A buying frenzy has sent the value of the total Bitcoin stock past $1.5bn and the price of a single Bitcoin has doubled in less than two weeks. Having passed $100 on April 1, it peaked at $147 in the small hours of Wednesday morning. Untethered to any real asset, the Bitcoin’s price is determined only by speculation on exchanges around the world […] (Financial Times, 3 April 2013)

- What can the literature tell us about why Bitcoins are used, what creates their value, and whether they are a bubble?
Bitcoin
$3,844.00
-2.88%▼
Households, How Many?

• Overlapping generations
  – Classic example
  – Time and number = infinity

1\textsuperscript{st} generation

2\textsuperscript{nd} generation

3\textsuperscript{rd} generation
OLG Model

- Tirole (1985), based on Diamond (1965)'s OLG model.
  - 2 period-lived agents. Work only in the first.
  - Population \( L_t = (1 + n)^t \)
  - One physical good per period.
  - CRS technology \( Y_t = F(K_t, L_t) = L_t f(k_t) \), operated by competitive firms:
    \[
    r_t = f'(k_t) \quad w_t = f(k_t) - k_t f'(k_t) = \phi(r_t)
    \]
- Program of agent born at \( t \)
  \[
  \max \quad u(c_{1t}) + \beta u(c_{2t+1})
  \]
  \[
  \text{s.t.} \quad c_{1t} + s_t = w_t
  \]
  \[
  c_{2t+1} = (1 + r_{t+1}) s_t
  \]
  implies a savings function \( s_t = s(w_t, r_{t+1}) \)
Equilibrium with no bubbles

- Asset market clearing:

\[ K_{t+1} = L_t s(w_t, r_{t+1}) \]

dividing by \( L_t \)

\[ (1+n)k_{t+1} = s(w_t, r_{t+1}) = s(w(k_t), r(k_{t+1})) \equiv S(k_t, k_{t+1}) \]

- Assume \( \frac{dk_{t+1}}{dk_t} = \frac{S_1}{1+n-S_2} \in [0,1] \): then we have Solow dynamics

![Figure 3.2](image)
Introduce $M$ pieces of paper. Let us look for an equilibrium in which these are valued at price $p_t$ each period.

The gross rate of return on bubbles is then $\frac{p_{t+1}}{p_t}$. No arbitrage with capital implies

$$1 + f'(k_{t+1}) = \frac{p_{t+1}}{p_t} \quad (2)$$

Write $B_t = Mp_t$ for the aggregate value of the bubble, and $b_t = \frac{B_t}{L_t}$. Then (2) implies

$$B_{t+1} = B_t \left(1 + f'(k_{t+1})\right) \quad \Rightarrow \quad b_{t+1} = b_t \left(\frac{1 + f'(k_{t+1})}{1 + n}\right)$$
Equilibrium with bubbles

- Savings can now be done in capital and bubbles:

\[ K_{t+1} + B_t = L_t s(w_t, r_{t+1}) \]

so

\[ k_{t+1} = \frac{s(k_t, k_{t+1}) - b_t}{1 + n} \tag{3} \]

the bubble reduces capital accumulation. We have \( k_t \geq 0 \) and assume free disposal of bubbles: \( b_t \geq 0 \).
Steady-state

- Write (3) in steady-state: \( b = s(k, k) - (1 + n)k \)
- On the other hand
  \[
  b_{t+1} = b_t \left( \frac{1 + f'(k_{t+1})}{1 + n} \right)
  \]
  implies that if \( b_t \neq 0 \) is to be constant, then we must have \( f'(k) = n \) at the steady-state
- Recall that with a neoclassical production function and no depreciation, the steady-state resource constraint is
  \[
  f(k) = c + k(1 + n) - k = c + nk
  \]
Steady-state per-capita consumption is maximal for the golden rule level of capital:
\[
 f'(k^*) = n
\]
Dynamic inefficiency

Moreover, for $k > k^*$, a decrease in steady-state capital increases steady-state per-capita consumption.

- the economy has “overaccumulated” capital. Productivity is insufficient to cover the resources used each period to provide the newborn with the current level of capital per person.

So, when $k > k^* \iff f'(k) < n$ the economy is **dynamically inefficient**: a Pareto-improvement can be reached by increasing the consumption of the current generation, reducing the stock of capital, and therefore increasing the consumption of all future generations.
Bubbles and dynamic inefficiency

- In steady-state: \( b = s(k, k) - (1 + n)k \). With bubbles, \( f'(k) = n \)

![Diagram showing steady state with bubbles](image)

**Figure 5.2**
Steady state of the Diamond model with bubbles

- *Always* one steady state with no bubbles (point A, capital \( k^d \))
- *If* \( f'(k^d) < n \), (dynamic inefficiency of the Diamond steady-state), another steady-state exists with a positively value bubble
Assume dynamic inefficiency. Let’s look at the “phase diagram” (this is not completely exact, since the dynamical system is discrete)

- $k_{t+1} - k_t = \frac{s(k_t, k_{t+1}) - b_t}{1 + n} - k_t \equiv g(k_t, b_t)$
- $b_{t+1} - b_t = \frac{b_t(f'(k_{t+1}) - n)}{1 + n} = \frac{b_t(f'(k_t + g(k_t, b_t)) - n)}{1 + n}$
Bubbles in OLG: conclusion

- Two types of paths can emerge:
  - The stable arm path: the economy converges to the bubbly steady-state, which satisfies the golden rule $f'(k^*) = n$
  - Paths in which a bubble exists in the transition, but not in the steady-state, which is the Diamond steady-state with $k^d$
  - (Paths with too large an initial bubble are ruled out, since they eventually imply capital decumulation exceeding the capital stock)

- Conclusion: in a dynamically inefficient economy:
  - Bubbles can exist as long as they are not too large
  - In a knife-edge case, they exist in steady-state and solve the dynamic inefficiency
Abel, Mankiw, Summers, and Zeckhauser (1989) propose a criterion for evaluating dynamic inefficiency that only involves comparisons of cash profits from capital, as a rate of return, with output coming from the history of the level of investment. They find that one cannot reject efficiency. Geerolf (2017) adjusts for land rent, taking it out, and adjusts profits of entrepreneurs, a fraction of which is arguably simply opportunity-cost wages and not a real return. He finds some economies may have been on inefficient paths. One notes in the reported results that some Asian miracle economies are among the excessive-investment economies.

Related and substantively important, in every monetary model there is also an equilibrium in which money does not have value due to self-fulfilling bad expectations. And economy without money is too efficient to support fiat, see Cass, Okuno, and Zilcha (1978), Green and Zhou (2005) and Levine (1989).
The value of money comes from cash-in-advance or payment of taxes

Models with money but having the requirement that money must be used.

There are other ways to give money an economic value. Suppose money must be used in trade or must be used to make certain required payments owed to the government. These arguments were given for fiat money but can be applied to tokens.

In an early paper on monetary economics, Starr (1974) specified, for tractability, a finite horizon model. This raised the problem that in the last period, money could not have value because nobody would want it – there is no future with it. However, money had a value, nevertheless, because agents were required to pay taxes to the government with it at the end. Thus, money was always demanded in equilibrium, and the price could not fall to zero. Some see this as a realistic setting and one reason why fiat monies, as compared with tokens, have value in actual economies. But see Bitcoin in Ohio.
Likewise, the so-called cash-in-advance model of Clower (1967) specifies that fiat money is legal tender, thus has to be acquired at least one period in advance in order to allow purchases with it in a given period. One could say that money has value in these models due to a legal stipulation requiring its use. See also Bryant and Wallace (1984). In the model of Lucas and Stokey (1983), some designated goods do not require cash in advance (the credit goods), but other goods do require cash in advance (the cash goods). Such models with credit goods allow real consumption loans and exchange in goods without money, but if there is at least a subset of crucial cash goods, however small, then the price of money cannot go to zero. Money must have a positive value in equilibrium.
A Hybrid Model of Positive Token Values

In sum, there are two ways of making money have value. Endogenously in an equilibrium given the underlying environment, and exogenously by legal or other restrictions. Layering the two ways of modeling money delivers endogenous valuation of tokens in realistic economic settings. In this way, we can talk about innovations in payment systems using tokens which, nevertheless, take as given and utilize valued fiat money. It is not an “either/or” proposition. In contrast, much of the literature thinks of Bitcoin, and other cryptocurrencies, as competing with fiat money.
Figure 1
The Turnpike Model
Problem 2:

\[
\max_{\{c^t_i\}_{t=0}^{\infty}, \{M^t_i\}_{t=1}^{\infty}, t=0} \sum_{t=0}^{\infty} \beta^t U(c^t_i)
\]

subject to

\[c^t_i \geq 0 \quad \text{all } t \geq 0\]
\[M^t_i \geq 0 \quad \text{all } t \geq 0\]
\[p_t c^t_i + M^t_{i+1} \leq p_t y^t_i + M^t_i - z^t_i \quad \text{all } t \geq 0\]

(5)

\[
- \frac{\beta^{t-1} U'(c^t_{i-1})}{p_{t-1}} + \frac{\beta^t U'(c^t_i)}{p_t} + \theta^t_i = 0 \quad \text{all } t \geq 1
\]

(6)

Thus,

\[
\frac{U'(c^t_{i-1})}{\beta U'(c^t_i)} \geq \frac{p_{t-1}}{p_t} \quad \text{all } t \geq 1
\]

(7)

where (7) must hold as an equality if \(M^t_i > 0\) and as an inequality if and only if \(\theta^t_i > 0\), that is, when the marginal utility of a unit of fiat money spent on period \(t-1\) consumption exceeds the marginal utility of a unit of fiat money spent on period \(t\) consumption and there is no more fiat money to spend in period \(t-1\).
**Definition.** A monetary equilibrium is a sequence of finite positive prices \(\{p_t^\ast\}_{t=0}^{\infty}\) and sequences of consumptions \(\{c_t^i\ast\}_{t=0}^{\infty}\), money balances \(\{M_t^i\ast\}_{t=0}^{\infty}\), and lump-sum taxes \(\{z_t^i\ast\}_{t=0}^{\infty}\) for each agent type \(i = A, B\) such that

- Maximization: the sequences \(\{c_t^i\ast\}_{t=0}^{\infty}\), \(\{M_t^i\ast\}_{t=1}^{\infty}\) solve Problem 2 relative to \(\{p_t^\ast\}_{t=0}^{\infty}\), \(\{z_t^i\ast\}_{t=0}^{\infty}\), and \(M_0^i\ast\).
- Market clearing: \(c_t^A\ast + c_t^B\ast = 1\), all \(t \geq 0\).

**Proposition 1.** No interior optimum \(\lambda\) can be supported in a monetary equilibrium without intervention, that is, with \(z_t^i\ast = 0\) for all \(i = A, B\).
PROPOSITION 4. There exists a noninterventionist monetary equilibrium with constant prices, with binding nonnegativity constraints on money balances in every other period, and with alternating consumption sequences.
Problem 1:

$$\max_{\{c_t^A\}_{t=0}^\infty, \{c_t^B\}_{t=0}^\infty} w^A \left[ \sum_{t=0}^\infty \beta^t U(c_t^A) \right] + w^B \left[ \sum_{t=0}^\infty \beta^t U(c_t^B) \right]$$

subject to (1) where $w^A > 0$, $w^B > 0$, $w^A + w^B = 1$. Necessary and sufficient first-order conditions for Problem 1 are

(2) \quad $w^t \beta^t U'(c_t) - \theta_t = 0$, $i = A, B$, $t \geq 0$

where the $\theta_t$ are positive Lagrange multipliers. Trivial manipulation of (2) yields

(3) \quad $\frac{U'(c_t^A)}{U'(c_t^A)} = \frac{U'(c_t^B)}{U'(c_t^B)}$ \quad all $t, \tau \geq 0$.

Conditions (1) and (3) are fully equivalent with

(4) \quad $c_t^A = \lambda$, \quad $c_t^B = 1 - \lambda$. \quad $0 < \lambda < 1$ \quad all $t \geq 0$. 
(See Figure 2.) Thus a necessary and sufficient condition for a feasible interior allocation \( \{c^A_t\}_{t=0}^\infty, \{c^B_t\}_{t=0}^\infty \) to be optimal is that each agent of type \( A \) receive \( \lambda \) units of the consumption good in each period \( t \). That this condition is necessary for optimality follows from the obvious fact that if condition (3) is not satisfied for some periods \( t \) and \( \tau \), then there is a Pareto superior feasible allocation.

**Figure 2**
Optimal Allocations in the Turnpike Model
PROPOSITION 2. Any interior optimum $\lambda$ with $\beta \leq \left[ \frac{\lambda}{1-\lambda} \right]$ and $\beta \leq \left[ \frac{(1-\lambda)\lambda}{1-\lambda} \right]$ can be supported in a monetary equilibrium with rate of deflation $1 - \beta$; with $z_{t}^{b*} = p_{t-1}^{*}[\lambda - \beta(1-\lambda)] \geq 0$ for $t \equiv 1$, $t$ odd, and $z_{t}^{b*} = 0$ otherwise; and with $z_{t}^{a*} = p_{t-1}^{*}[(1-\lambda) - \lambda\beta] \geq 0$ for $t \equiv 2$, $t$ even, and $z_{t}^{a*} = 0$ otherwise.

PROPOSITION 3. Any monetary equilibrium with nonbinding nonnegativity constraints on money balances on each agent in each period supports an optimal allocation and hence requires some intervention.