

The Eventual Failure of Price Fixing Schemes

ROBERT M. TOWNSEND*

*Graduate School of Industrial Administration, Carnegie-Mellon University,
Pittsburgh, Pennsylvania 15213*

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This essay examines the feasibility of a government buffer stock program which attempts to fix a relative price over time. In the model, m risk averse agents with random endowments of two commodities maximize expected utility over an infinite horizon. It is shown that a price fixing scheme will fail with probability 1, regardless of the price set by the government and regardless of the initial level of buffer stocks. The proof of this proposition turns on some well-known properties of random walks.

1. INTRODUCTION

In actual economies, uncertainty as to the direction and extent of future price movements is pervasive. This has led some policymakers to argue that government programs which attempt to fix prices could reduce uncertainty and increase economic welfare. This argument has been particularly prevalent in policy discussions concerning the stabilization of raw material and agricultural commodity prices and the stabilization of exchange rates of internationally traded currencies. With regard to commodity price stabilization, Keynes [6] argued for the establishment of an ever-normal granary to eliminate the violence of raw material price fluctuations associated with an unregulated competitive system. Other economists respond with the long-held belief that prices play a crucial role in allocating resources efficiently. Waugh [10] and Oi [7] have even argued that price instability may be beneficial, though Samuelson [8] showed that the fluctuations proposed in a partial equilibrium setting were not really feasible in a closed model. With regard to

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currency price stabilization, the debate over fixed versus flexible exchange rates has had a long history. The argument has centered in particular on whether speculation is or is not destabilizing and on whether there is or is not an adequate hedge against price uncertainty (see, for example, [1, 9]).

To some extent confusion on these issues stems from the failure of economists to provide coherent stochastic models. Many economic models are deterministic and hence give the impression that uncertainty is an anomaly rather than a fact of life. As for stochastic models, many are partial equilibrium and hence leave open the question of whether the uncertainty can be eliminated. This essay takes a modest step toward the formulation of a coherent general equilibrium model in which one can analyze price fixing schemes.

The model adopted is a pure exchange economy in which the endowments of two goods, (x) and (y) , are stochastic. In each period the aggregate endowments of (x) and (y) are independently distributed, and each series is assumed to be independent and identically distributed over time. Both goods can be stored with no storage costs and no depreciation. Acting competitively, each of m risk averse individuals maximizes expected utility over an infinite horizon by choice of the amount of each good to consume in each period and the amount to carry over to the following period. The rate at which (y) exchange for (x) in each period is the price upon which the analysis focuses. It is supposed that the government attempts to maintain a fixed price by its willingness to exchange (y) for (x) with the public at a specified rate. In general such a policy requires that the government maintain buffer stocks of both goods.

For the specific model examined it turns out that regardless of the price set by the government and regardless of the initial level of government buffer stocks, a price fixing scheme fails in finite time with probability one. Equivalently, in the same model without government, maximizing individuals in competitive markets will not act in such a way as to fix the relative price over time. The relative price does play a crucial role in the allocation of resources over time. For example, with random aggregate endowments, there will be eventually a succession of realizations with the aggregate endowment of (y) low relative to the aggregate endowment of (x) . In competitive markets without government the relative price of (y) would rise, thereby discouraging its consumption. If the relative price of (y) were fixed at its average value or less, the government's stocks of (y) would be depleted. In short, there must be some price flexibility; with the relative price fixed the government must absorb all disturbances in relative supply with its buffer stocks, yet it is not capable of doing so.

This essay proceeds as follows. Section 2 describes the structure of the model and analyzes the decision problem of the individual with a fixed relative price. The properties of individual storage and consumption decisions

are then shown to be inconsistent with any initial level of government stocks and with any fixed price. Formal proofs are contained in Section 3. Section 4 presents some concluding remarks.

2. STRUCTURE AND PROPERTIES OF THE MODEL

The stochastic nature of the endowments is now made precise. In each time period there is a finite set of states Ω . Let P be a probability measure with $0 < P(\omega)$ for each $\omega \in \Omega$. Let X_t and Y_t denote the economy's total endowment of (x) and (y) , respectively, at time t . X_t and Y_t are assumed to be independent, nonnegative random variables. Moreover, X_t and Y_t are each independent and identically distributed over time. It is also assumed that there exists some $\bar{\omega} \in \Omega$ such that $X_t(\bar{\omega}) = Y_t(\bar{\omega}) = 0$. Let $X_t = E(X_t) + \epsilon_{xt}$ and $Y_t = E(Y_t) + \epsilon_{yt}$, where ϵ_{xt} and ϵ_{yt} are each symmetric and take on values in the integer lattice.

Each individual is assumed to have a fixed and constant proportion of the aggregate endowment of each good, δ_x^j and δ_y^j being the j th individual's share of X_t and Y_t , respectively. Let $Z_t^j = [\delta_x^j X_t, \delta_y^j Y_t]$. Let $K_{x,t}^j$ and $K_{y,t}^j$ denote the j th individual's stocks of (x) and (y) , respectively, at the end of the period t , to be chosen during period t . Let X_t^j and Y_t^j denote the j th individual's consumption of (x) and (y) , respectively, at time t . Let R_t denote the relative price of (y) in terms of (x) at time t . Let $r_t' = [1, R_t]$. Then, regarding as parameters current and future prices $\{R_t; t = 1, 2, \dots\}$ and initial stocks $\{K_{x,0}^j \geq 0, K_{y,0}^j \geq 0\}$, the objective of individual j is to maximize $E_0 \sum_{t=1}^{\infty} \beta^{t-1} U^j(X_t^j, Y_t^j)$ with respect to $\{K_{x,t}^j, K_{y,t}^j, X_t^j, Y_t^j, t = 1, 2, \dots\}$ subject to the constraints:

- (i) $X_t^j + R_t Y_t^j \leq K_{x,t-1}^j + R_t K_{y,t-1}^j + r_t' Z_t^j - K_{x,t}^j - R_t K_{y,t}^j, \quad t = 1, 2, \dots,$
- (ii) $X_t^j \geq 0; \quad Y_t^j \geq 0, \quad t = 1, 2, \dots,$
- (iii) $K_{x,t}^j \geq 0; \quad K_{y,t}^j \geq 0, \quad t = 1, 2, \dots.$

Here $0 < \beta < 1$, and $E_t(\cdot)$ denotes the expectation conditioned on all realizations up to and including time t . Constraint (i) is the budget constraint for individual j at time t ; to be noted is the imposed absence of borrowing possibilities and forward contracts. Constraint (ii) implies that expenditures at time t must be nonnegative. Constraint (iii) states that the stocks of each good must be nonnegative.

$U^j(\cdot, \cdot)$ is assumed to have the following properties:

- (i) $U^j(\cdot, \cdot) = g^j[H^j(\cdot, \cdot)]$, where $g^j(\cdot)$ is of class C^2 with first derivative $g_1^j(\cdot) > 0$ and second derivative $g_{11}^j(\cdot) < 0$. Also, $g^j(0) = 0, g_1^j(H) \rightarrow 0$ as $H \rightarrow \infty$, and $g_{11}^j(H) \rightarrow \infty$ as $H \rightarrow 0$.

(ii) $H^j(\cdot, \cdot): R_+^2 \rightarrow R_+^1$ is of class C^2 with strictly positive first partial derivatives. Also, $H_{11}^j(\cdot, \cdot) < 0$ with the bordered Hessian of $H^j(\cdot, \cdot)$ positive.

(iii) $H^j(\cdot, \cdot)$ is homogeneous of degree 1.

(iv) $H_1^j(X_t^j, Y_t^j) \rightarrow \infty$ as $(X_t^j, Y_t^j) \rightarrow (0, \bar{Y})$ for each $\bar{Y} > 0$, $H_2^j(X_t^j, Y_t^j) \rightarrow \infty$ as $(X_t^j, Y_t^j) \rightarrow (\bar{X}, 0)$ for each $\bar{X} > 0$.

Thus $U^j(\cdot, \cdot)$ is a positive monotonic transformation of a homogenous function of degree 1 and is said to be homothetic. $U^j(\cdot, \cdot)$ possesses properties (ii) and (iv), among others.

Properties of individual storage decisions are now derived on the assumption that the government maintains a fixed price for all time. The assumption that price will remain fixed for all time does not eliminate uncertainty from the decision problem of the individual; endowments are still stochastic. Yet, each individual need be concerned only with the total value of stocks of (x) and (y) in terms of one of the goods, say (x) . The exchange of (y) for a predetermined amount of (x) in any future period is guaranteed by the government. The individual's decision problem may be viewed in two stages. In a given period and state, the individual has available a known amount of savings in terms of (x) of the previous period and the realized value of his endowment in the specified state. He decides on his optimal amount of savings in terms of (x) , and consequently on his current expenditures in terms of (x) . With this timing problem solved, the individual then chooses his current consumption of (x) and (y) at the specified price.

To state this more formally, the following notation is needed. Let \bar{R} denote the relative price of (y) in terms of (x) fixed by the government. Then, by assumption, $R_t = \bar{R}$ for all $t \geq 1$. Let $r' = [1, \bar{R}]$. Let $K_t^j = K_{x,t}^j + \bar{R}K_{y,t}^j$. Let $I_t^j = X_t^j + \bar{R}Y_t^j$. Then, by the strict monotonicity of preferences, $I_t^j = K_{t-1}^j + r'Z_t^j - K_t^j$. By virtue of the separability of the objective function over time, maximizing choices of X_t^j and Y_t^j can be expressed as functions of I_t^j and \bar{R} only. Let $h_x^j(I_t^j, \bar{R})$ and $h_y^j(I_t^j, \bar{R})$ denote maximizing choices of X_t^j and Y_t^j , respectively, for the function $U^j(X_t^j, Y_t^j)$ subject to the constraint that $X_t^j + \bar{R}Y_t^j = I_t^j$. Then an indirect utility function $V^j(\cdot, \cdot)$ is defined by $V^j(I_t^j, \bar{R}) = U^j[h_x^j(I_t^j, \bar{R}), h_y^j(I_t^j, \bar{R})]$. Upon substitution, given $K_0^j \geq 0$, the objective of individual j is to maximize $E_0 \sum_{t=1}^{\infty} \beta^{t-1} V^j(K_{t-1}^j + r'Z_t^j - K_t^j, \bar{R})$ with respect to $\{K_t^j; t = 1, 2, \dots\}$ subject to the constraints:

$$(i) \quad K_{t-1}^j + r'Z_t^j - K_t^j \geq 0, \quad t = 1, 2, \dots,$$

$$(ii) \quad K_t^j \geq 0, \quad t = 1, 2, \dots$$

Constraint (ii) is apparently weaker than the restriction that $K_{x,t}^j$ and $K_{y,t}^j$ each be nonnegative. However, the imposition of the latter constraint would not alter the optimal storage rules for $\{K_t^j; t = 1, 2, \dots\}$.

From the homogeneity of $H^j(\cdot, \cdot)$, the demand functions $h_x^j(\cdot, \bar{R})$ and $h_y^j(\cdot, \bar{R})$ are linear. It follows that $V^j(\cdot, \bar{R})$ is strictly concave.¹ Clearly, $V^j(\cdot, \bar{R})$ is of class C^1 and strictly increasing. From property (i) of $U^j(\cdot, \cdot)$, $V_1^j(I, \bar{R}) \rightarrow \infty$ as $I \rightarrow 0$, and $V_1^j(I, \bar{R}) \rightarrow 0$ as $I \rightarrow \infty$.

Given a fixed relative price, the individual's stochastic dynamic programming problem is equivalent to a choice between consumption and savings in a one-good model in which future income is subject to random shocks. Foley and Hellwig [4] and Brock and Mirman [2] have established existence and analyzed optimal consumption policies for similar problems, and hence the crucial properties of the solution to the problem of this essay are given here without proof.² Let $W_t^j = K_{t-1}^j + r'Z_t^j$ denote the wealth of individual j at time t in terms of (x) . Let $\tilde{K}_t^j = \varphi^j(W_t^j, \bar{R})$ denote a maximizing choice of stocks K_t as a function of W_t^j and \bar{R} . Also, let $M^j(\bar{R}) = \max_{\omega \in \Omega} r'Z_t^j(\omega)$. Then the following properties can be established:

- (i) For every $\bar{R} > 0$ there exists a unique $K_*^j(\bar{R})$ such that

$$\varphi^j[K_*^j(\bar{R}) + M^j(\bar{R}), \bar{R}] = K_*^j(\bar{R}) > 0.$$

Also, $\varphi^j(0, \bar{R}) = 0$.

- (ii) For every $\bar{R} > 0$ let $A^j(\bar{R}) = [0, K_*^j(\bar{R})]$. Then if $\tilde{K}_t^j \in A^j(\bar{R})$, $\tilde{K}_{\tau}^j \in A^j(\bar{R})$ for all $\tau > t$.

- (iii) For all $K_0^j \geq 0$, $\text{prob} \{\tilde{K}_t^j > K_*^j(\bar{R}) \text{ i.o.}\} = 0$.

As \tilde{K}_t^j eventually enters the set $A^j(\bar{R})$ with probability 1, it may be assumed $K_0^j \in A^j(\bar{R})$. Then \tilde{K}_t^j is bounded above by $K_*^j(\bar{R})$ for all t .

Given the existence of a solution to the individual's optimization problem when the government is maintaining a fixed price, there will be determined some $\{I_t^j; j = 1, 2, \dots, m; t = 1, 2, \dots\}$. Given I_t^j , individual j will purchase utility maximizing quantities of (x) and (y) at the fixed price \bar{R} . For the policy to be feasible it is specified that \bar{R} be an equilibrium price in the sense that any excess demand for (x) by individuals be matched by an excess supply on the part of the government. It is now shown that no price can be maintained indefinitely far into the future, regardless of the initial level of government stocks.

The proof of this proposition turns on associating with each fixed price

¹ $V^j(\cdot, \bar{R}) = g^j[\tilde{V}^j(\cdot, \bar{R})]$, where $\tilde{V}^j(\cdot, \bar{R})$, the indirect utility function for $H(\cdot, \cdot)$, is linear. Strict concavity of $V^j(\cdot, \bar{R})$ then follows from the strict concavity of $g^j(\cdot)$. Note also that this implies that $U^j(\cdot, \cdot)$ is strictly concave. Let D denote the bordered Hessian of $U^j(\cdot, \cdot)$. It can be shown that $V_{11}^j = (U_{11}^j U_{22}^j - U_{12}^j U_{21}^j)/(-D)$. By property (ii), $D > 0$ and so the numerator is positive. Also, $U_{11}^j = g_{11}^j(H_1^j)^2 + g_1^j H_{11}^j < 0$.

² The problem solved by Foley and Hellwig [4] has a two-point distribution of income in each period, but the generalization to the distribution of this essay is straightforward.

\bar{R} set by the government a pure exchange, nonstochastic, no-storage economy for which \bar{R} is a competitive equilibrium price. In some sense, the most difficult price for which to prove the infeasibility of price fixing is the competitive equilibrium price for the model with endowments $E(X_t)$ and $E(Y_t)$ for (x) and (y) , respectively. Such a price corresponds to the competitive equilibrium price of the "average" economy in the model without storage but with random endowments. For this price, properties of a random walk in R^2 with zero drift are used. Roughly speaking, with probability 1 there will be runs over a number of periods in which there are smaller than average endowments of (y) and higher than average endowments of (x) . Total incomes will be at their average values, and agents will wish to consume average amounts of (y) at the fixed price. But (y) is in short supply, and eventual failure is inevitable. For all other prices, properties of random walks with nonzero drift are used; if the government sets a price of (y) which is in some sense too low, there will be an excess supply of (x) .

More formally, consider a static pure exchange economy with fixed endowments. Each agent j has the same utility function $U^j(\cdot, \cdot)$ of the stochastic model with storage. Also let $Z^j = [\delta_x^j X, \delta_y^j Y]$ denote the endowment of agent j , where the share parameters δ_x^j and δ_y^j are the same as in the stochastic model with storage. It is assumed that the $U^j(\cdot, \cdot)$ and Z^j , $j = 1, 2, \dots, m$, are such that (x) and (y) are gross substitutes. Then for $X > 0$ and $Y > 0$ there exists a unique competitive equilibrium price $R^* = \pi(X, Y)$, where $\pi(\cdot, \cdot) : (0, \infty)^2 \rightarrow R_+^1$. The following properties of $\pi(\cdot, \cdot)$ are established in Section 3.

LEMMA 1. $\pi(\cdot, \cdot)$ is continuous.

LEMMA 2. $\pi(X, \bar{Y}) \rightarrow 0$ as $X \rightarrow 0$ for each $\bar{Y} > 0$. $\pi(\bar{X}, Y) \rightarrow \infty$ as $Y \rightarrow 0$ for each $\bar{X} > 0$.

These then yield the following

PROPOSITION. Given any fixed price $\bar{R} \in [0, \infty)$ and any initial level of government stocks of (x) and (y) , there exists with probability 1 some $T^* < \infty$ at which those stocks will be insufficient to maintain the fixed price.

The proposition has the obvious

COROLLARY. If there exists a competitive equilibrium without government with relative price R_t^* at time t , then there does not exist some constant \bar{R} such that $R_t^* = \bar{R}$ for all t .

It might be argued that the expected time of failure of the price fixing scheme is infinite and that this mitigates the conclusions of the proposition. However, the proposition asserts that failure occurs with probability 1 in finite time.

3. FORMAL PROOFS

Proof of Lemma 1. Let $J(X, Y, R) = \sum_{j=1}^m [h_x^j(\delta_x^j X + \delta_y^j Y R, R) - \delta_x^j X]$. Then $R^* = \pi(X, Y)$ is chosen so that $J[X, Y, R^*] = 0$. By properties (i) and (iv) of $U^j(\cdot, \cdot)$ and the implicit function theorem, $h_x^j(\delta_x^j X + \delta_y^j Y R, R)$ is of class C^1 with respect to X, Y , and R . Hence, the first partial derivatives of $J(\cdot, \cdot, \cdot)$ are all continuous with respect to X, Y , and R . As (x) and (y) are gross substitutes, $J_3(\cdot, \cdot, \cdot) > 0$. Hence, the implicit function theorem applies and $\pi(\cdot, \cdot)$ is continuous. Q.E.D.

Proof of Lemma 2. The proof is by contradiction. Suppose that for some $\bar{Y} > 0$, $\pi(X, \bar{Y}) \rightarrow 0$ as $X \rightarrow 0$. Then there exists a sequence $\{X_n\}$ such that $X_n \rightarrow 0$ as $n \rightarrow \infty$ and for which the corresponding sequence of equilibrium prices, $\pi(X_n, \bar{Y})$, is bounded from below by some $R'' > 0$. Then as (x) and (y) are gross substitutes, $J[X_n, \bar{Y}, \pi(X_n, \bar{Y})] \geq J[X_n, \bar{Y}, R'']$. But

$$\lim_{n \rightarrow \infty} J[X_n, \bar{Y}, R''] = \sum_{j=1}^m h_x^j[\delta_y^j \bar{Y} R'', R''] > 0.$$

With $J[X_n, \bar{Y}, \pi(X_n, \bar{Y})] = 0$, this establishes the desired contradiction. By a symmetric argument, $\pi(\bar{X}, Y) \rightarrow \infty$ as $Y \rightarrow 0$ for each $\bar{X} > 0$. Q.E.D.

Proof of Proposition. There are four cases to be considered.

Case (i). $\bar{R} = 0$.

By property (ii) of $U^j(\cdot, \cdot)$, agents are never satiated with respect to consumption of (y). Hence if $\bar{R} = 0$, government stocks of (y) will be depleted in the first period of operation.

Case (ii). $\bar{R} = \pi[E(X_t), E(Y_t)]$.

Let G_0 denote the stock of (y) held by the government at $t = 0$, A_t denote the cumulative net amount of (y) sold to individuals by the government in exchange for (x) up through and including time t , and Y_t^* denote the aggregate of (y) over all individuals made available for consumption from private wealth in period t . Also, let $S_{yT} = \sum_{t=1}^T \epsilon_{yt}$, $S_{xT} = \sum_{t=1}^T \epsilon_{xt}$, and $S_T = [S_{xT}, S_{yT}]$. Then S_T is a random walk in R^2 and has the property that for any integers B_x and B_y there exists some $T^* < \infty$ such that $S_{xT^*} = B_x$ and $S_{yT^*} = B_y$ with probability one (see [5, p. 796]). For the proof let B_y be the largest integer less than or equal to $-G_0 - 1 - \sum_{j=1}^m (K_*^j \bar{R})$, and let B_x be such that $\sum_{j=1}^m h_y^j(\delta_x^j B_x + \delta_y^j B_y \bar{R} - K_*^j) \geq 0$. This last choice is possible by the linearity of $h_y^j(\cdot, \bar{R})$. In period t the government must sell $\sum_{j=1}^m h_y^j(I_t^j, \bar{R}) - Y_t^*$ units of (y) to the public. Then the cumulative net sales of (y) at time T^* will be

$$A_{T^*} = \sum_{t=1}^{T^*} \left[\sum_{j=1}^m h_y^j(I_t^j, \bar{R}) - Y_t^* \right] = \sum_{j=1}^m h_y^j \left(\sum_{t=1}^{T^*} I_t^j, \bar{R} \right) - \sum_{t=1}^{T^*} Y_t^*,$$

using the linearity of $h_y^j(\cdot, \bar{R})$. Also,

$$\sum_{t=1}^{T^*} Y_t^* \leq \sum_{t=1}^{T^*} Y_t + \sum_{j=1}^m (K_*^j/\bar{R})$$

and

$$\sum_{t=1}^{T^*} I_t^j \geq \sum_{t=1}^{T^*} (\delta_w^j X_t + \delta_y^j Y_t \bar{R}) - K_*^j.$$

From these two inequalities and by substitution one obtains the inequality

$$\begin{aligned} A_{T^*} &\geq \sum_{j=1}^m \{h_y^j[\delta_w^j E(X_t) T^* + \delta_w^j S_{xT^*} + \delta_y^j E(Y_t) \bar{R} T^* + \delta_y^j S_{yT^*} \bar{R} - K_*^j, \bar{R}]\} \\ &\quad - E(Y_t) T^* - S_{yT^*} - \sum_{j=1}^m (K_*^j/\bar{R}) \\ &= T^* \left\{ \sum_{j=1}^m h_y^j[\delta_w^j E(X_t) + \delta_y^j E(Y_t) \bar{R}, \bar{R}] - E(Y_t) \right\} \\ &\quad + \sum_{j=1}^m h_y^j[\delta_w^j B_x + \delta_y^j B_y \bar{R} - K_*^j, \bar{R}] - B_y - \sum_{j=1}^m (K_*^j/\bar{R}). \end{aligned}$$

But by the choice of \bar{R} .

$$\sum_{j=1}^m h_y^j[\delta_w^j E(X_t) + \delta_y^j E(Y_t) \bar{R}, \bar{R}] - E(Y_t) = 0.$$

Hence by the choice of B_x and B_y , $A_{T^*} \geq G_0 + 1 > G_0$.

Case (iii). $0 < \bar{R} < \pi[E(X_t), E(Y_t)]$.

$\pi(\cdot, \cdot)$ is continuous and $\pi[X, E(Y_t)] \rightarrow 0$ as $X \rightarrow 0$. Hence, by the intermediate value theorem there exists some X^* , $0 < X^* < E(X_t)$, such that $\bar{R} = \pi[X^*, E(Y_t)]$. Let $\gamma = E(X_t) - X^*$. Then $X_t = X^* + (\gamma + \epsilon_{xt})$ with $E(\epsilon_{xt} + \gamma) > 0$. Let $S_{xT} = \sum_{t=1}^T (\epsilon_{xt} + \gamma)$ and let $S_{yT} = \sum_{t=1}^T \epsilon_{yt}$. As in Case (ii) let B_y be the largest integer less than or equal to $-G_0 - 1 - \sum_{j=1}^m (K_*^j/\bar{R})$. Then by Feller [3, pp. 202-203], S_{xt} visits $(-\infty, a)$ a finite number of times for all $a > 0$. Hence, there exists some T^* such that $S_{yT^*} = B_y$ and S_{xT^*} is large enough that $\sum_{j=1}^m h_y^j(\delta_w^j S_{xT^*} + \delta_y^j S_{yT^*} \bar{R} - K_*^j, \bar{R}) \geq 0$. Then as in Case (ii)

$$\begin{aligned} A_{T^*} &\geq T^* \left\{ \sum_{j=1}^m h_y^j[\delta_w^j X^* + \delta_y^j E(Y_t) \bar{R}, \bar{R}] - E(Y_t) \right\} \\ &\quad + \sum_{j=1}^m h_y^j(\delta_w^j S_{xT^*} + \delta_y^j S_{yT^*} \bar{R} - K_*^j, \bar{R}) - S_{yT^*} - \sum_{j=1}^m (K_*^j/\bar{R}) > G_0. \end{aligned}$$

Case (iv). $\pi[E(X_t), E(Y_t)] < \bar{R} < \infty$.

By a proof quite similar to Case (iii), initial stocks of (x) will be insufficient to maintain the fixed price. Q.E.D.

Proof of Corollary. Let $G_0 = 0$ and apply the proof of the proposition. Q.E.D.

4. CONCLUDING REMARKS

In the model the government attempts to fix a relative price over time by its willingness to exchange two commodities with the public at a specified rate. It has been shown that under specified assumptions such a policy fails with probability 1. Yet there remains the question of whether there exist feasible price fixing schemes based on endowment taxation. One particularly simple scheme based on commodity taxation also fails. For suppose that the government were to attempt to maintain the price corresponding to Case (ii) of the proposition by confiscation (restitution) of the surfeit (deficit) of X_t over $E(X_t)$ in each period, and similarly for (y) . Note that if such a policy were successful there would be no storage by individuals. Then the cumulative net amount of (x) confiscated by the government through period T would be $\sum_{t=1}^T \epsilon_{xt}$, and the stated properties of random walks imply that government stocks eventually will be insufficient to maintain the fixed price. Other fixed prices fail similarly. However, it is difficult to establish the properties of more complicated price fixing schemes based on endowment taxation.

The failure of the most obvious price fixing schemes raises some questions concerning the existence and properties of a competitive equilibrium in the model without government. It can be shown that the set of economies which possess competitive equilibria without government is nonempty; hence the result of this essay is not vacuous. In particular, if all agents have identical preferences and endowments, then there exists a solution to the stochastic dynamic programming problem of the representative individual. (It can be noted in passing that this autarkic solution has the property that the implicit distribution of prices displays some smoothness as compared with the distribution which would prevail if storage by individuals were prohibited.) Yet the question of existence is a difficult one in a model of any generality.

Given existence, the properties of the competitive equilibrium path without government could be analyzed. If conventional results are any guide, it might be shown that the path is Pareto optimal. This then would raise some doubt concerning the desirability of even feasible government buffer stock programs. Yet these conjectures are beyond the scope of the present essay.

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