Who Sees the Trades? The Effect of Information on Liquidity in Inter-dealer Markets

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Abstract

Dealers, who strategically supply liquidity to traders, are subject to both liquidity and adverse selection costs. While liquidity costs can be mitigated through inter-dealer trading, individual dealers’ private motives to acquire information compromise inter-dealer market liquidity. Post-trade information disclosure can improve market liquidity by counteracting dealers’ incentives to become better informed through their market-making activities. Limited disclosure, however, exacerbates the adverse selection problem in inter-dealer markets, in turn decreasing equilibrium liquidity provision. A non-monotonic relationship may arise between the fraction of dealers that receive post-trade information and market liquidity. This points to a practical concern: a strategic post-trade platform has incentives to maximize adverse selection and may choose to release information in a way that minimizes equilibrium liquidity provision.

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1 Introduction

A large volume of financial transactions occur in decentralized markets. Decentralization creates a role for intermediaries, called dealers, to offer liquidity by making markets. Dealers are subject to two main sources of risk. First, they must manage liquidity costs associated with large net positions that arise from inventory holdings and regulatory compliance. Second, they run the risk of facing informed trades, bringing rise to adverse selection.

Liquidity costs arise when dealers accumulate positions through trades with their respective clients, whether it be institutional investors or periphery dealers. Dealers often attempt to reduce liquidity costs by engaging in offsetting inter-dealer trades. As a consequence, these markets exhibit a tiered trading structure, where dealers make markets for clients, taking into account future opportunities to unwind positions through trades with other dealers.\textsuperscript{1} The extent to which dealers successfully unwind their positions is complicated by the fact that, through their individual market-making activities, dealers becoming differentially and asymmetrically informed.

Little is known about how information asymmetries affect overall market liquidity in decentralized asset markets with a tiered trading structure. Of particular interest is the release of trading information that results from decentralized market-making activity. This information can be aggregated through clearing platforms or trade repositories or may even be made public through the transparency of a trading platform. Our analysis focuses on how differences in the availability of post-trade information from the market-making stage in a dealer-intermediated market impacts overall market liquidity. We consider exogenous information disclosure policies first and then examine what might result from the strategic sale of post-trade information.

We analyze a model in which agents are randomly bilaterally matched and given an opportunity to trade. At the center of our model are dealers, who make markets for traders. Trade occurs in two stages: the first stage involves trade between dealers and traders (the “market-making stage”) and the second stage

\textsuperscript{1}This tiered trading structure has been widely documented across various asset classes, including corporate bonds (Di Maggio, Kermani and Song (2017)), municipal bonds (Li and Schürhoff (2019)), derivatives (Peltonen, Scheicher and Vuilleme (2014)), federal funds (Bech and Atalay (2010)), and other inter-bank markets (Craig and Von Peter (2014)).
involves trade between dealers (the “inter-dealer market”). In the market-making stage, dealers quote a bid-ask spread at which they are willing to purchase or sell the asset to the traders. Traders decide whether to buy or sell from a dealer. Dealers who purchase an asset from a trader, referred to as “long dealers”, accumulate excess inventory while dealers who sell assets, referred to as “short dealers”, seek to replenish their inventory. Traders have private values for the asset that are dispersed around the true common value. Hence, dealers who are successful in their market-making activity learn something about the true value of the asset. Each dealer’s own trading activity does not provide enough information for them to determine the true value with certainty. However, observing the full set of trades of all dealers from the market making stage would perfectly reveal the value of the asset.

With no disclosure of post-trade information, we show that inter-dealer markets arise endogenously and inter-dealer trading achieves better allocations, even with two-sided asymmetric information problem between dealers. At play are strategic complementarities and substitutability: greater inter-dealer liquidity increases dealers’ incentives to provide liquidity in the market-making stage, but private incentives to acquire more information than other dealers curtail dealers’ equilibrium provision of liquidity to traders. With full post-trade information disclosure, dealers offer greater liquidity provision in the market-making stage. Providing post-trade information to all dealers reduces their incentive to extract information about the true value of the asset by quoting larger bid-ask spreads in the market-making stage. This results in tighter spreads as greater liquidity provision yields greater profits.

Improvements in market liquidity achieved through disclosure translate to greater welfare for all agents. Disclosure effectively reduces negative externalities that limit inter-dealer market liquidity. When dealers can mitigate liquidity costs more effectively through inter-dealer trading, they find it profitable to increase their liquidity provision to traders. In this sense, increased liquidity implies greater efficiency, as it implies that dealers facilitate better allocations.\(^2\)

Limited disclosure of information arises when some dealers are allowed access to post-trade information while others are not. It also arises if all dealers have ac-

\(^2\)We discuss the relation between market liquidity and efficiency in greater detail in Appendix B.
cess to post-trade information, but some dealers can process that information faster than others. Limited disclosure can harm market liquidity by exacerbating the information problem that naturally arises in inter-dealer markets. To illustrate this in a tractable setting, we consider a setup where, after the market-making stage, a fraction of dealers observe the net positions. It is common knowledge how many dealers become informed, but the identities of those who are informed are kept secret. The availability of post-trade information to a subset of dealers has two opposing effects. When dealers are collectively informed, inter-dealer liquidity can improve, and dealers provide greater liquidity in the market-making stage. However, individually, dealers who ex-post become informed may forgo off-setting trades in favor of informed inter-dealer trades. When the fraction of dealers that will become informed is known to be low, the second effect overpowers the first, causing market liquidity to decreases. In this case, inventory imbalances across dealers worsen, as some informed dealers trade with, instead of against, their position. This outcome flips when the share of dealers who become informed is sufficiently high.

The fact that limited disclosure of information can be worse than either full disclosure or no disclosure is potentially problematic in practice, because a post-trade platform might strategically choose to release information in a socially inefficient manner. We conclude our analysis by considering a strategic post-trade platform that sells information to maximize profits. We find an equilibrium in which only a fraction of dealers become informed. In particular, our equilibrium supports the sale of information to the fraction of dealers that corresponds to the worst possible outcome in the exogenous limited disclosure setting. This occurs because the platform has an incentive to increase adverse selection as this makes the information it is selling more valuable.

The implications of post-trade disclosure are particularly relevant in light of recent technological innovations that have the potential to transform current market infrastructure. As private and public market infrastructure providers alike explore options to replace legacy technology, a pointed opportunity arises to redesign the way in which trading and post-trade platforms operate. One important direction of potential adoption is distributed ledger technology (DLT). One inherent advantage of DLT is its conducive nature to implement precise and flexible distribution of information to members of a network, through a permissioned
structure.

We highlight two relevant insights with respect to this issue. First, the hazardous effects of limited disclosure highlight the importance of ubiquitous access by all relevant market participants. Second, at the heart of inefficiencies arising from strategic platform is the inability of a profit-maximizing service provider to ex-ante commit to a disclosure policy that maximizes liquidity. In this respect, a DLT-based platform that makes it costly, if not impossible, to change its disclosure policy without market consensus could strictly improve the provision of liquidity, and ultimately the allocation of assets.

Our paper is the first to consider the implications of information disclosure in a OTC market in which agents’ endogenous information acquisition leads to an asymmetric information problem. Past studies have focused either on considering the asymmetric information problems that arise with endogenous information acquisition (e.g. Glode, Green and Lowery (2012), Vanasco (2017)), or how the disclosure environment affects trading (e.g. Pagano and Röell (1996), Pagano and Volpin (2012)). Our position is that it is essential to consider these two issues (acquisition of private information and disclosure of information) in conjunction in order to fully understand how agents’ incentives to acquire information are affected by the disclosure environment. Incorporating these two components is challenging – we accomplish this by de-emphasizing other important considerations in OTC trading, including cost of inventory and trade size (e.g. Colliard, Foucault and Hoffmann (2018)), dealer competition (e.g. Lester, Rocheteau and Weill (2015)), and network structure (e.g. Wang (2017), Sambalaibat (2018)), that have been addressed elsewhere in the literature.

The remainder of the paper is organized as follows. Section 2 discusses the related literature. Section 3 introduces the model. In Section 4, we solve the equilibrium without disclosure of post-trade information. Section 5 considers the setting with full disclosure of post-trade information. Section 6 considers the setting with limited disclosure of post-trade information. In Section 7 we consider a strategic platform that endogenously chooses the post-trade information structure. We conclude in Section 8. All Proofs are in Appendix A.
2 Related Literature

Early work on information disclosure in financial markets considered full transparency versus opaque markets. See, for example, the theoretical work of Glosten and Milgrom (1985), Madhavan (1995) and de Frutos and Manzano (2005) and the experimental work of Flood, Huisman, Koedijk and Mahieu (1999) and Bloomfield and O’Hara (2015). These studies considered repeated interaction between traders and one or more specialists (market makers) and focused on the price formation process.

Our paper contributes to the growing literature on liquidity provision in decentralized markets. This literature seeks to explain how market liquidity is impacted by search frictions and other aspects of the decentralized trading process (see, for example, Duffie, Gârleanu and Pedersen (2005) and Lagos and Rocheteau (2009)), the interaction of OTC markets and the primary credit market (Arseneau, Rappoport and Vardoulakis (2017)), and policies that reduce informational asymmetries (Cujean and Praz (2016)). Our work contributes to the stream on informational asymmetries and, in particular, focuses on informational asymmetries about the common value of the asset to the dealers that arise endogenously from trading outcomes in the market-making stage, before the inter-dealer market takes place. In our environment, dealers engage in bilateral bargaining in the inter-dealer market, and complementing existing studies where inter-dealer market are competitive, such as Duffie et al. (2005), Lagos and Rocheteau (2009), and Dunne, Hau and Moore (2015).

Our emphasis is on the impact of informational asymmetries that carry over from the market-making stage into the inter-dealer market. The fact that informational asymmetries amplify the adverse selection problem in financial markets and hence can lead to reduced trade has been highlighted in other contexts. Glode et al. (2012) show how informational asymmetries between a buyer and seller in an OTC market can result in a break-down in trade during times of high price volatility. Vanasco (2017) shows that screening efforts taken by an asset originator to improve the quality of an asset can result in a less liquid market for the asset due to the informational asymmetries this creates between the originator and the investors. Di Maggio and Pagano (2018) look at informational asymmetries that arise from differences in information processing costs of participants.
in the secondary market for an newly issued asset, and show that this results in reduced trading and lower prices. Finally, Lester, Shourideh, Venkateswaran and Zetlin-Jones (2019) examine the impact of asymmetric information and imperfect competition in an OTC market where informed sellers are randomly matched with one or both of the uniformed buyers. They find that reducing informational asymmetries is welfare improving only when buyers have a lot of market power and the adverse selection problem is relatively severe.

We complement other studies that examine the effects of transparency on financial markets. Pagano and Volpin (2012) studies how transparency of asset-backed securities at issuance affects secondary market liquidity. Pagano and Röell (1996) examines the impact of transparency on various market settings. Cujean and Praz (2016) look at private information regarding inventories and examine the impact of a policy to make these inventories public. They consider a one-period model with OTC trade between investors and do not consider market making activities of dealers.

Our finding that making post-trade information from the market-making stage public before the inter-dealer market takes place leads to narrower bid-ask spreads and hence increased liquidity in the market-making stage is consistent with empirical studies on market transparency. Bessembinder, Maxwell and Venkataraman (2006), Edwards, Harris and Piwowar (2007), and Bessembinder and Maxwell (2008) examine the introduction of the Transaction Reporting and Compliance Engine (TRACE) for the US corporate bond market in July 2002. Under this program, transaction data related to all trades in publicly issued corporate bonds was made available to the public. These studies all found that the implementation of TRACE led to reductions in bid-ask spreads and increased liquidity, with some exceptions for thinly traded bonds or very large trades. Benos, Payne and Vasios (2016) examine the impact of the Dodd-Frank trading mandate that required US persons to trade interest rate swaps on Swap Execution Facilities (SEF) with open limit order books. They found that the introduction of SEF trading led to economically significant improvement in liquidity. Boehmer, Saar and Yu (2005) examined trading on the New York Stock Exchange. They found that effective spreads of trades decline following the introduction of the OpenBook policy in January of

\footnote{Formally, they examine variations in a parameter that defines the level of precision of signal on counterparty inventory.}
2002 that provided limit-book order information to traders off the exchange floor. Finally, in regards to CDS markets, Loon and Zhong (2016) show that the liquidity improves for index CDS contracts following the introduction real time reporting and public dissemination of OTC swap trades on December 31, 2012.

3 Model

Consider a market where an asset is traded bilaterally. There is a measure 1 of dealers, indexed $i \in [0, 1]$ and a measure 1 of traders, indexed by $j \in [0, 1]$. All agents are risk-neutral. Trading occurs in two stages. In the first stage (“market-making”), dealers and traders are matched at random. Dealers “make markets” by offering bid-ask prices to the traders with whom they are matched. In the second stage (“inter-dealer”), dealers are randomly matched with other dealers with whom they have an opportunity to trade. This two-stage structure is intended to capture the tiered trading structure common in decentralized dealer markets.\(^4\)

Market-Making Stage. At $t = 1$, each dealer is matched with one trader. The asset has a common value $v$ to all dealers that equals $\bar{v} + x$ or $\bar{v} - x$ with equal probability, for some $x > 0$. Each trader $j$ has a private value for the asset $v_j$ that is drawn independently from a uniform distribution with support $[v - D, v + D]$, for some $D > 0$. The magnitude of $D$ captures the dispersion in traders’ private values of the asset.

Each dealer makes an ultimatum bid-ask offer $P_i = (P_i^b, P_i^a)$, where $P_i^b$ represents the bid price, at which the trader can sell the asset to the dealer, and $P_i^a$ represents the ask price at which the trader can purchase the asset from the dealer.\(^5\) Given a dealer’s set of bid-ask prices $P_i$, a trader $j$ chooses whether to accept the bid price, accept the ask price, or reject the dealer’s offer. Formally, a trader $j$ matched to dealer $i$ chooses an action $\gamma_j \in \{\text{accept} P_i^b, \text{accept} P_i^a, \text{reject}\}$.

\(^4\)There is considerable empirical evidence that dealer intermediated markets have a tiered structure (For example, see Li and Schürhoff (2019), Afonso, Kovner and Schoar (2013), Craig and Von Peter (2014)). To keep the model tractable, we take this structure as given and focus on the strategic behavior of dealers to endogenize market liquidity. For papers that endogenize the two-stage structure, see Viswanathan and Wang (2004) or Neklyudov (2014).

\(^5\)Empirical studies find that dealers exercise substantial bargaining power (Green, Hollifield and Schürhoff (2006)).
to maximize her payoff, which can be written as

$$1 \{ \text{accept } P^b_i \} \cdot (P^b_i - v_j) + 1 \{ \text{accept } P^a_i \} \cdot (v_j - P^a_i) \geq 0,$$

where $1 \{ \cdot \}$ is an indicator function for the trader’s action. Hence, a trader $j$ chooses the action

- accept $P^a_i$ if $v_j \geq P^a_i$
- accept $P^b_i$ if $P^b_i \geq v_j$
- reject otherwise.

We limit our attention to the case in which dealers offer a symmetric bid-ask spread around $\bar{v}$, such that $P_i = (P^b_i, P^a_i) = (\bar{v} - \delta_i, \bar{v} + \delta_i)$ for some $\delta_i > 0$.

In Figure 1, $\bar{v}$ represents a dealer’s expected value of $v$ before trading. The top line illustrates the distribution of trader values when the actual value of $v$ is $\bar{v} - x$. The bottom line illustrates the distribution of trader values when the actual value of $v$ is $\bar{v} + x$. The red shaded regions represent the mass of traders, under each realization of $v$, who are willing to accept a bid offer (to the left of $\bar{v} - \delta$) or an ask offer (to the right of $\bar{v} + \delta$).

An important insight revealed by Figure 1 is that if $v = \bar{v} - x$, then the likelihood that a trader will accept the dealer’s bid price is high compared to the likelihood that a trader would accept the ask price. Conversely, if $v = \bar{v} + x$, then a trader is more likely to accept the dealer’s ask price than the bid price.

At the end of $t = 1$, dealers who have purchased the asset have a net position
of 1 and we refer to them as “long dealers.” Dealers who have sold the asset have a net position of −1 and we refer to them as “short dealers.” Finally dealers who did not trade have a net position of 0 and we refer to them as “neutral dealers.” We use $\theta \in \{l, s, n\}$ to denote the type of the dealer at the end of $t = 1$.

**Inter-Dealer Market.** At $t = 2$ the inter-dealer market opens. All dealers are randomly bilaterally matched. Within each pair, one dealer is picked at random and allowed to make an ultimatum offer to his or her counterparty. Both dealers have equal probability of being picked. The dealer that makes the ultimatum offer is called the “offering” dealer and the counterpart is the “receiving” dealer.

An offering dealer $i$ of type $\theta$ makes an offer $(\sigma_{i, \theta}, P_{d, i, \theta})$, where $\sigma_{i, \theta} \in \{buy, sell, no trade\}$ indicates the actions that the offering dealer wants, and $P_{d, i, \theta}$ denotes the transaction price. A receiving dealer $i$ who receives offer $(\sigma_{i, \theta}, P_{d, i, \theta})$ from dealer $\neq i$ makes a decision of whether to accept or reject the offer. Formally, $\gamma_{i, \theta}(\sigma_{i, \theta}, P_{d, i, \theta}) \in \{accept, reject\}$.

**Post trade.** At the end of $t = 2$, after all trade occurs, a dealer with a net position of $k \in \{-2, -1, 0, 1, 2\}$ of the asset incurs a cost $\Delta|k|$, where $\Delta \in \left(\frac{D}{\sqrt{2} + 1}, D\right)$. This cost can be motivated in a number of ways. One natural interpretation is that $\Delta$ represents the per-unit opportunity cost of providing collateral to a central clearer. In many over-the-counter markets, a central counter party (CCP) helps to reduce counterparty risk between market participants. Over the course of the day, CCP members report their trades to the CCP. At the end of the day, the CCP calculates the net position of each member and asks members to provide contributions that are proportional to the net positions. Another interpretation comes from the fact that we assume that any dealer that finishes stage 2 in a long or short position must immediately unwind this position by selling or buying (respectively) the asset at a price equal to its true value. It is reasonable to assume this would be done through an intermediary who charges a per-unit inventory cost.

**Equilibrium.** The solution concept is Perfect Bayesian Equilibrium. Given an information structure, an equilibrium consists of dealers’ market-making offer strategies $\delta_i^*$, dealers’ inter-dealer market offer strategies $(\sigma_{i, \theta}, P_{d, i, \theta})$, and dealers’

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6The specific form of the inter-dealer offer, while tractable, is without loss of generality.
trade decisions given offers in the inter-dealer market, traders’ trade decisions
given offers in the market-making stage, and dealers’ and traders’ beliefs. We
look for symmetric equilibrium strategies such that $\delta^*_i = \delta^*_k$ for $\forall i, k$. Formally:

**Definition 1.** A Perfect Bayesian Equilibrium is dealers’ market-making strategies \{\delta^*_i\}_i and inter-dealer offer strategies \{(\sigma^*_i, P^d_i)\}_i;\theta=n,l,s, dealers trading strategies conditional on inter-dealer offers \{\gamma^*_i(\sigma^*_0, P^d_0)\}_i, traders’ trading strategies conditional on bid-ask offers \{\gamma^*_j(P^b, P^a)\}_j, and traders’ beliefs and dealers’ beliefs such that:

1. dealer i’s market making strategies $\delta^*_i$ maximize the dealer’s expected profits at $t = 1$, and inter-dealer offer strategies and \{(\sigma^*_i, P^d_i)\}_\theta=n,l,s trading strategies $\gamma^*_i(\sigma^*_0, P^d_0)$ maximize the dealer’s conditional expected payoff at $t = 2$;
2. trader j’s trading strategy $\gamma^*_j(P^b, P^a)$ maximizes her payoffs at $t = 1$;
3. dealers’ and traders’ beliefs are consistent with Bayes’ Rule.

4 Opaque market

We begin by analyzing our two-stage decentralized market assuming that in-
formation regarding trades in the market-making stage remains private. This is
the baseline case from which alternative assumptions on information disclosure
and a strategic model of the sale of trading information are explored.

4.1 Market-Making Strategies

In the market-making stage at $t = 1$, each dealer $i$ offers a bid-ask offer $P_i = (\bar{v} - \delta_i, \bar{v} + \delta_i)$ corresponding to some spread $\delta_i$ at which he offers to buy and sell an asset from a trader. In addition to determining profits conditional on trade, a dealer’s spread impacts: (1) the likelihood that a trader accepts her offer to trade, and (2) the dealer’s posterior belief about the asset value $v$ conditional on a trader accepting his offer. Acceptance of offers fully reveal the value of the asset if $D < x$. We focus on interesting cases where $D > x$. In such an environment it is useful to make a distinction between the two sets of offers that the dealer can make:

**Definition 2** (Market-making strategies). Dealer $i$ is said to employ a
• **partially revealing offer** if he chooses $\delta_i \in (0, D - x)$;

• **fully revealing offer** if he chooses $\delta_i \geq D - x$.

**Partially revealing offers.** To begin, we restrict our attention to partially revealing offers, i.e. when $\delta_i < D - x$. Recall, as outlined in Section 3, that a trader accepts a dealer’s bid offer if and only if her valuation $v_j$ is less than $\bar{v} - \delta_i$, and accepts a dealer’s ask offer if and only if $v_j$ is greater than $\bar{v} + \delta_i$. It is straightforward to see that a trader is willing to accept at most one of the offers, for any $\delta_i > 0$. The likelihood that dealer $i$’s bid offer $\bar{v} - \delta_i$ is accepted is given by

$$P(v = \bar{v} + x) \cdot P(\bar{v} - \delta_i > v_j | v = \bar{v} + x) + P(v = \bar{v} - x) \cdot P(\bar{v} - \delta_i > v_j | v = \bar{v} - x) = \frac{D - \delta_i}{2D}.$$  

Following a similar computation, the likelihood that a dealer $i$’s ask offer $\bar{v} + \delta_i$ is accepted is also given by (1). Note that as $\delta_i$ increases, the likelihood that a trader accepts a dealer’s offer monotonically decreases. Since a greater spread is associated with a less attractive offer to a trader, fewer traders are willing to accept the dealer’s offers.

A trader’s valuation $v_j$ is centered around the common value $\bar{v}$. As a result, dealer $i$, who is initially uninformed about $v$, revises his beliefs concerning the common value $\bar{v}$ conditional on an offer being accepted by a trader. This implies that a dealer can directly affect how much he learns from market-making through his bid-ask offer strategy $P_i$. Specifically, choosing a wider bid-ask spread reduces the probability that the dealer trades, as noted above, but also provides more information about the value of $\bar{v}$ conditional on a trade. We now formalize this second effect.

We can characterize dealer $i$’s interim beliefs regarding $\bar{v}$ conditional on successfully trading with a trader. Conditional on dealer $i$’s bid offer $\bar{v} - \delta_i$ being accepted, dealer $i$’s belief on the expected value of $\bar{v}$ is given by

$$P(v = \bar{v} + x | \bar{v} - \delta_i > v_j) \cdot (\bar{v} + x) + P(v = \bar{v} - x | \bar{v} - \delta_i > v_j) \cdot (\bar{v} - x) = \bar{v} - \frac{x}{D - \delta_i} \cdot x.$$  

Likewise, conditional on dealer $i$’s ask offer $\bar{v} + \delta_i$ being accepted, dealer $i$’s belief on the expected value of $\bar{v}$ is given by $\bar{v} + \frac{x}{D - \delta_i} \cdot x$. 

First, consider how the dealer, when employing partially revealing offers, affects the revision of his beliefs regarding \( v \) conditional on trading. Since \( \frac{x}{D-\delta_i} \) increases in \( \delta_i \), a higher \( \delta_i \) leads to a greater downward revision on the expected value of \( v \), conditional on a bid offer being accepted. Correspondingly, a higher \( \delta_i \) leads to a greater upward revision in the expected value of \( v \), conditional on an ask offer being accepted.

![Figure 2: Impact of increasing bid-ask spreads](image)

Each line represents traders’ valuations conditional on \( v \). The top line corresponds to \( v = \tilde{v} - x \) and the bottom line corresponds to \( v = \tilde{v} + x \). An increase in the bid-ask spread from \( \delta \) to \( \delta' \) corresponds to a smaller likelihood of trade, represented by the teal shaded regions.

This effect is illustrated in Figure 2. Suppose a trader accepts an ask offer. If the dealer had chosen a tight bid-ask spread (i.e. \( \delta \)), then Figure 2 suggests that the probability that \( v = \tilde{v} + x \) is twice as large as the probability that \( v = \tilde{v} - x \). If, instead, the dealer had chosen a wider bid-ask spread (i.e. \( \delta' \)), then conditional on an ask offer being accepted, the probability that \( v = \tilde{v} + x \) becomes considerably more than twice as large as the probability that \( v = \tilde{v} - x \). This is reflected in the fact that the set of private value traders that would accept the ask offer shrinks disproportionately more in the low asset value state than in the high asset value state. Figure 2 also shows that the probability of any offer being accepted is smaller when the bid-ask spread is wider.

**Fully revealing offer.** Dealers can also choose to employ a fully revealing offer strategy, in which case \( \delta_i \geq D - x \). Given traders’ optimal trading strategies, it is straightforward to see that if the dealer sets \( \delta_i \geq D - x \), then any accepted offer fully reveals the state of nature. For example, if the state is \( v = \tilde{v} + x \), then \( v_j \) can
only be smaller than the bid price $\bar{v} - \delta_i \leq \bar{v} - D + x$ if $v_j \leq \bar{v} - D + x$, which is not possible since $v_j \in [v - D, v + D]$. So a bid offer can only be accepted when the state is $v = \bar{v} - x$. A similar argument shows that an ask offer can only be accepted if the state is $v = \bar{v} + x$. As a consequence, the dealer becomes perfectly informed about the common value $v$ through trade.

Figure 3: Trading under fully revealing market-making offers

Each line represents traders’ valuations conditional on $v$. The top line corresponds to $v = \bar{v} - x$ and the bottom line corresponds to $v = \bar{v} + x$. The red shaded regions represent the traders who are willing to accept an offer. For bid-ask spread $\delta > D - x$ a bid or ask offer is accepted by a trader only when $v = \bar{v} - x$ or $v = \bar{v} + x$, respectively.

Figure 3 illustrates the case of a fully-revealing offer. If $\delta$ is sufficiently large, bid offers are only accepted if $v = \bar{v} - x$ and ask offers are only accepted if $v = \bar{v} + x$. In the case of a fully revealing offer, the likelihood that dealer $i$’s bid offer $\bar{v} - \delta_i$ is accepted is given by the probability that the state is $v = \bar{v} - x$ multiplied by the probability that $\bar{v} - \delta_i > v_j$ in that state. Since each state of the world occurs with equal probability, this can be written as

$$P(v = \bar{v} - x) \cdot P(\bar{v} - \delta_i > v_j | v = \bar{v} - x) = \frac{D + x - \delta_i}{4D}. \quad (3)$$

Note that (3) is equal to (1) if $\delta_i = D - x$. By symmetry, the likelihood that a dealer $i$’s ask offer $\bar{v} + \delta$ if accepted is also given by (3). As in the case of a partially revealing offer, the probability of a trade decreases as $\delta$ increases.

In general, dealers face a clear trade-off between acquiring more information through trade, and increasing the likelihood of trade. With partially revealing offers, dealers become better informed through trade, but still remain uncertain about the underlying common value $v$. With fully revealing offers, dealers learn
perfectly the underlying state of the world, conditional on a trade, but trade with a lower likelihood.

4.2 Inter-dealer Markets

Inter-dealer trading depends on dealers’ collective market making strategies, since these strategies determine the share of short, neutral, and long dealers in the inter-dealer market. In this section we study dealers participation in inter-dealer markets and how inter-dealer market liquidity relates to dealers’ liquidity provision at \( t = 1 \).

Given our focus on perfect Bayesian equilibrium we need to examine the sub-game that arises at \( t = 2 \), conditional on some symmetric market making strategy \( \delta_i = \hat{\delta} \) assumed to be employed by dealers at \( t = 1 \). We start by analyzing the case where dealers use partially revealing offers at the market making stage; \( \hat{\delta} < D - x \). Notice that dealers enter inter-dealer trading with dispersed beliefs regarding \( v \), even if they chose the same market-making spread \( \delta_i = \hat{\delta} \), because dealers update their beliefs about \( v \) conditional on a trade being accepted.

We assume that dealers do not know the position of the dealers with whom they are matched. Without loss of generality, we use a long dealer as an example to illustrate the strategic considerations in inter-dealer trading. The dealer’s offer must take into account that his counterparty could have a long, short, or neutral position. By making an offer to sell, a long dealer could offset his position, and avoid liquidity cost \( \Delta \). However, he may instead prefer to increase his long position if it is more profitable.

First, consider when the long dealer makes an offer to sell, which would offset his position. Suppose that receiving dealers infer that a sell offer is only made by a long dealer, i.e. in equilibrium the long dealer separates from other types by signaling through his offer. The reservation prices of a long, neutral, and short

\footnote{When \( \hat{\delta} \geq D - x \), all dealers who trade are on the same side of the market (all are either long or short) and there is no inter-dealer trading.}
A dealer’s reservation price is comprised of two parts. The first depends on a receiving dealer’s belief about the expected value of \( v \) conditional on trade. Given receiving dealers’ (correct) beliefs that a sell offer is made by a long dealer, they require the reservation price to reflect the expected value of \( v \) conditional on that, and their private information.

The second depends on whether the trade provides a netting benefit to the receiving dealer. For any offer received, a dealer incurs an (additional) \( \Delta \) cost if his net position increases as a result of accepting the offer. Hence, in the above case, where dealers receive an offer to sell, with the exception of short dealers, a receiving dealer requires an additional \( \Delta \) subtracted from the price. Short dealers, who gain from netting their \(-1\) position, are willing to pay a premium of \( \Delta \).

The set of reservation prices consists of the three candidate prices at which a long dealer may want to make a sell offer. When a long dealer signals his type by making a sell offer, the counterparty with whom he can make the most profitable trade is a short dealer. A short dealer’s reservation is the highest due to the netting benefits and because a short dealer’s valuation of the asset is greatest conditional on trade.

Lowering the price to another dealer type’s reservation price increases the likelihood of trade, but it is not profitable. Just as a receiving dealer’s beliefs adjusted to account for the likelihood that he was matched to a long dealer, a long dealer accounts for the likelihood that he was matched to a particular dealer. As such, conditional on a specific dealer type pair, both parties of an inter-dealer trade form identical beliefs about \( v \). This reveals a powerful insight: when all dealers are differentially but equally informed, i.e. \( \delta_i \) is identical, then surplus from trade only occurs when a dealer trades with a counterparty with an opposite position.

More generally, if, in equilibrium, separation is to occur through inter-dealer offers between dealer types, then a dealer’s payoff maximizing offer is set at the
reservation price of a dealer of a opposite position. In the case of a neutral dealer, no trades yield a positive surplus.

So far, we have focused on a long dealer’s optimal sell offer strategy. Would he instead want to make a buy offer? By making a buy offer, a long dealer increases his net position, which would be associated with an additional $\Delta$ liquidity cost. In addition, a long dealer’s private information works against him – his valuation of $v$ is lower relative to other dealer types. It is straightforward to verify that there does not exist a feasible price at which buy offers for a long dealer are profitable. Building on this, we can fully characterize the inter-dealer subgame given $\delta_i = \hat{\delta} < D - x$ for all $i$.

**Lemma 1 (Inter-dealer Trading).** Suppose that all dealers execute symmetric partially-revealing offers at $t = 1$. Then, in inter-dealer markets:

1. short dealers make offer $(\text{buy}, \bar{v} - \Delta)$ and only accept offers $(\text{sell}, P^d)$ for $P^d \leq \bar{v} + \Delta$;

2. long dealers make offer $(\text{sell}, \bar{v} + \Delta)$ and only accept offers $(\text{buy}, P^d)$ for $P^d \geq \bar{v} - \Delta$;

3. neutral dealers do not make any offers, and reject all offers.

Interestingly, the price at which dealers make offers is independent of $\hat{\delta}$. Even though dealers are asymmetrically informed about each other’s type, a trade uniquely identifies the type of the offering and receiving dealers’ types. Since successful trades entail matches between dealers of opposite positions, their beliefs ex-post offset each other – conditional on trading, the expected value of $v$ is exactly $\bar{v}$ for both parties.

What remains is to characterize inter-dealer markets where all dealers choose $\hat{\delta} > D - x$. As highlighted in Section 4.1, when a dealer uses a fully revealing offer in the previous period, he is able to infer the true value of $v$. When all dealers use fully revealing offers, all dealers who successfully trade acquire the same position, depending on $v$. Specifically, all dealers who trade become short or long dealers, if $v = \bar{v} + x$ or $\bar{v} - x$, respectively. Consequently, no offsetting trades can occur in inter-dealer markets. In short, there do not exist any inter-dealer trades that result in positive surplus when $\hat{\delta} > D - x$. 

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We have established that inter-dealer trading occurs if and only if partially revealing market-making strategies are chosen by dealers and we have characterized equilibrium inter-dealer trade for this situation. It is worth noting that efficient inter-dealer trading entails minimizing the total dead-weight loss associated with the nonzero positions held by dealers and this occurs in the equilibrium we describe where every match between a long and short dealer results in successful trade and no trade occurs otherwise.

4.3 Equilibrium With and Without Inter-dealer Trading

We now complete our equilibrium characterization by establishing when dealers make partially revealing offers in the market-making stage and when they do not. Equilibria with partially revealing offers and inter-dealer trading exist when the level of common value uncertainty, \( x \), is sufficiently small. There are two factors at play. First, dealers face a risk of failing to offload their position in the inter-dealer market, which brings rise to a winner’s curse problem. This risk increases with the amount of uncertainty surrounding the true asset value, as reflected by the magnitude of \( x \). Second, dealers individually do not internalize the value of inter-dealer market liquidity, which determines the extent to which gains from trade arise through netting positions. As a result, given other dealers’ market-making strategies, an individual dealer potentially has an incentive to choose a larger bid-ask spread, since he can reap the benefits of inter-dealer liquidity provided by other dealers and also be better informed about \( v \) in the inter-dealer trading stage. As \( x \) shrinks, the incentive to free ride on the liquidity provision of others, by deviating to a larger spread, gets smaller. Both factors are alleviated when \( x \) is below a cutoff, which we denote by \( x^{\text{trade}} \). Then, there exists a symmetric equilibrium with partially revealing offers that results in inter-dealer trading.

**Theorem 1** (Equilibrium with inter-dealer trade). Suppose that \( x < x^{\text{trade}} \) for some cutoff \( x^{\text{trade}} > 0 \). Then, there exists an equilibrium with bid-ask spread \( \delta^* = \frac{2D^2 + x^2 + \Delta D}{4D - \Delta} \in [0, D - x) \) and inter-dealer trade.

When \( x \) is sufficiently large, the gains to partially revealing offers are reduced and dealers may seek to buy assets solely for the purpose of capturing surplus from traders with extreme private values. In particular, when \( x \) is greater than a
threshold, which we denote by \( x^{\text{notrade}} \), there exists an equilibrium in which dealers make fully revealing offers that are accepted by traders who are willing to pay a high premium for liquidity and dealers are fully insured from the realization of \( x \).

**Theorem 2 (Equilibrium without inter-dealer trade).** Suppose that \( x > x^{\text{notrade}} \) for some cutoff \( x^{\text{notrade}} > 0 \). Then, there exists an equilibrium with bid-ask spread \( \delta^{**} = x + \frac{D + \Delta}{2} > D - x \) and no inter-dealer trading.

Theorems 1 and 2 lay out how the underlying uncertainty surrounding \( x \) relates to the nature (or lack) of inter-dealer trading. An equilibrium with inter-dealer trading exists if uncertainty about the common value is not too large (ie, \( x < x^{\text{trade}} \)) because lower uncertainty surrounding the common asset value mitigates the potential for the winner’s curse and reduces the incentive to free ride. In contrast, an equilibrium without inter-dealer trading exists when \( x \) is sufficiently large (ie, \( x > x^{\text{notrade}} \)) because more uncertainty surrounding the common asset value disincentivizes the selection of a partially revealing strategy absent the possibility of inter-dealer profits. Under certain conditions (see Appendix C) \( x^{\text{notrade}} < x^{\text{trade}} \) and hence equilibria with and without inter-dealer trading can coexist. Hence we have that for \( x < x^{\text{notrade}} \), only an equilibrium with inter-dealer trading exists, for \( x \in (x^{\text{notrade}}, x^{\text{trade}}) \), both types of equilibria exist and for \( x > x^{\text{trade}} \) only an equilibrium with market segmentation exists.

The coexistence of an equilibrium with inter-dealer trading and an equilibrium with segmentation for intermediate values of \( x \in [x^{\text{notrade}}, x^{\text{trade}}] \) sheds light on a potential channel through which inter-dealer markets, and more generally, OTC market liquidity are fragile. In this region, dealers’ expectations of the existence of inter-dealer trading pivotally determine their market-making strategies, which validate their beliefs. This self-fulfilling nature of market liquidity reveals a vulnerability of dealer intermediated markets to collective miscoordination by dealers.

In addition, a small uncertainty shock (i.e. increase in \( x \)) around \( x^{\text{trade}} \) may lead to a sudden breakdown in the inter-dealer markets. For example, inter-dealer trading, which accounts for 61 percent of all trades in Sterling OTC markets, fell to 2 percent during the Sterling flash crash on October 2016. In a report on the flash crash, the Financial Conduct Authority cites the sharp withdrawal of dealers
from inter-dealer markets as one of the key catalysts of the dramatic illiquidity episode.

### 4.4 Equilibrium Liquidity Provision By Dealers

Two aspects can be used to characterize equilibrium liquidity provisions by dealers. First, the equilibrium bid-ask spread captures the liquidity extended to traders by dealers who make markets. As an extension, we can relate $\delta$ to a direct measure of liquidity. Let $\mu(\delta)$ be the measure of traders that accept a dealer’s offer at $t = 1$. In an equilibrium with inter-dealer trading, we get

$$
\mu(\delta^*) = \frac{2(D - \Delta)D - x^2}{(4D - \Delta)D}.
$$

(5)

In an equilibrium with market segmentation, $\mu$ is given by

$$
\mu(\delta^{**}) = \frac{D - \Delta}{4D}.
$$

(6)

In both cases, $\mu$ increases in $D$ and decreases in $\Delta$. Suppose the condition provided in appendix C holds, so that $x^{notrade} < x^{trade}$. Then, for $x$ in the interval $(x^{notrade}, x^{trade})$, the two types of equilibria coexist. It is easy to verify that in such cases, equilibria inter-dealer trading vastly improve market liquidity.

**Corollary 1.** For $x \in (x^{notrade}, x^{trade})$, dealer liquidity provision with inter-dealer trading is greater than it is without inter-dealer trading.

In our setting, market liquidity and efficiency is greater when the private value $D$ is large and uncertainty $x$ is small. The parameter space in which inter-dealer markets are active is broadly consistent with other studies that study the relative efficiency of centralized and decentralized markets. For instance, Viswanathan and Wang (2004) compares one-shot and sequential auctions and shows that sequential trading is more efficient when customer orders are less informed. Glode and Opp (2017) endogenizes dealer information acquisition and further find that decentralized trading is more efficient relative to an auction when motives to trade are driven by private values.
5 Market Liquidity and Post-Trade Information Disclosure

The analysis in the previous section reveals how dealers’ private gains from becoming better informed limit equilibrium inter-dealer market liquidity, and potentially break down inter-dealer trading altogether. Lower inter-dealer market liquidity in turn lowers dealers’ liquidity provision incentives, and ultimately lowers efficiency. This suggests that efficiency can be improved by limiting the private benefits from being better informed in inter-dealer markets. We demonstrate how market liquidity can be improved upon through post-trade information disclosure.

Full Disclosure of Information. Consider the following extension to the model presented in Section 2. Suppose that in the beginning of period 2 and prior to matching between dealers taking place, information regarding the set of trades that occurred between traders and dealers at stage 1 is made public.\(^8\) At the extreme, we could assume that information regarding the direction (i.e. buy or sell) and price of each individual trade is made public. However it is sufficient in our setting to assume that the public observes the aggregate shares of bids and asks that were accepted in the market-making stage.

As a precursor, note that observing the outcome of all trades is sufficient to perfectly infer the underlying value of \(x\).\(^9\) This implies that the release of post-trade information from stage 1 makes all dealers informed about the true asset value at the beginning of stage 2. We now characterize the inter-dealer game, given full disclosure of information, conditional on all dealers having chosen some \(\hat{\delta} < D - x\):

Lemma 2 (Inter-dealer Trading under Full Disclosure). Suppose that all dealers execute partially-revealing offers at spread \(\hat{\delta}\) at \(t = 1\). Then, in inter-dealer markets:

\(^8\)This could occur, for example, if all trades in stage 1 are cleared through a CCP.

\(^9\)The dealers’ Bayesian updating problem is equivalent to determining which of two 3-sided coins was used to determine a sequence (or, in this case, measure) of observations: the high-value coin has the outcomes, B, N and S, with probabilities \(h\), \(m\) and \(l\), respectively and the low-value coin has the outcomes, B, N and S, with probabilities \(l\), \(m\) and \(h\), respectively; where \(h + m + l = 1\) and \(h > l\). Observing a large number of flips of a selected coin reveals the type of the coin with a level of precision that increases in the number of observations and in the size of the difference between \(h\) and \(l\). As the number of observations becomes infinite, the precision goes to 1 for any given values of \(h\) and \(l\).
1. short dealers make offer (buy, $v - \Delta$) and only accept offers (sell, $P^d$) for $P^d \leq v + \Delta$;

2. long dealers make offer (sell, $v + \Delta$) and only accept offers (buy, $P^d$) for $P^d \geq v - \Delta$;

3. neutral dealers do not make any offers, and reject all offers.

Inter-dealer trading under disclosure is similar to that without disclosure, as outlined in Lemma 1. The key difference is the prices at which dealers transact. Since all dealers are ex-post perfectly informed about $v$, the price reflects the true value $v$ plus the gains from trade $\Delta$. Dealer strategies are again fully separating: trade occurs exclusively between long and short dealers, who gain by offsetting each others’ positions. Naturally, this implies that, as in the case without information disclosure, inter-dealer trading with disclosure is efficient.

The primary effect of information disclosure is the elimination of strategic behavior aimed at increasing profits by extracting more information (via bigger bid ask spread) in the market-making stage. Since information is ensured to be available ex-post regardless of liquidity provision, information disclosure shuts down incentives to learn more through market-making. Specifically, since the offer fully reflects $v$, information rents are equal to zero. By shutting down the information rent, individual dealers’ incentives to deviate are weakened. This has two effects. First, all else equal, disclosure decreases equilibrium bid-ask spreads. Second, an equilibrium with inter-dealer trading exists for a larger interval of $x$.

We characterize the resulting equilibrium where dealers use partially revealing offers in the following:

**Theorem 3 (Full Disclosure).** Suppose there is full disclosure of stage 1 trade information and that $x < x^{\text{trade}, \text{disclosure}}$ for some cutoff $x^{\text{trade}, \text{disclosure}} > 0$. Then, there exists an equilibrium with bid-ask spread $\delta^{***} = \frac{(2D + \Delta)D}{4D - \Delta} \in [0, D - x)$ and inter-dealer trade. Furthermore, $x^{\text{trade}, \text{disclosure}} > x^{\text{trade}}$.

As in the case of no disclosure of post-trade information from stage 1 (Theorem 1) there is a restriction on the level of the common value of uncertainty $x$ that permits profitable market making. The relationship $x^{\text{trade}, \text{disclosure}} > x^{\text{trade}}$ is established in Appendix A.2. Moreover, $\delta^{***} < \delta^*$ (from Theorem 1). Consequently,
under full disclosure of information, market liquidity is enhanced. This improvement in liquidity is primarily driven by shutting down individual dealer’s strategic incentive to marginally increase bid-ask spread in the market making stage, thereby obtaining an information advantage in inter-dealer trading. Since dealers know that everyone will be informed about the aggregate state in the inter-dealer trading stage, regardless of their liquidity provision strategy, individual dealers choose to offer tighter spreads that maximize profits, irrespective of other dealers’ strategies. Consequently, we the following:

**Corollary 2.** Market liquidity is (weakly) greater under full disclosure of post-trade information than it is under no disclosure.

Corollary 2 points to a channel through which increased disclosure of post-trade information can be used to improve market liquidity. Namely, by eliminating the possibility of asymmetric information between dealers at the inter-dealer market, all dealers can ex-ante more aggressively offer liquidity to traders without strategic considerations with respect to becoming more informed than future counterparties.

### 6 Limited Access To Post-Trade Information

We have established that market efficiency increases with full disclosure of information regarding aggregate trading in the market-making stage. However, there are compelling reasons why information disclosure may be limited across participants in the inter-dealer market. Even if post-trade information is publicly available, dealer inattention may bar some dealers from incorporating relevant post-trade information in time for subsequent trades with other dealers. Dealer heterogeneity may lead to some dealers being able to process certain information better than others, which may also lead to dispersion in informedness.\(^\text{10}\) Finally, the supplier of post-trade information may choose to disclose information to only a subset of the dealers. We expand on this last point in Section 7, by introducing a profit-maximizing clearing platform that endogenously chooses the disclosure environment.

\(^{10}\)This idea is formalized by ?, in a setting where speculators have lower information processing costs than hedgers in the secondary market for an newly issued asset.
Limited Disclosure of Information. To analyze limited access to post-trade information we generalize the post-trade environment considered in Section 4 as follows. Suppose that prior to inter-dealer trading at $t = 2$, the vector of net positions of dealers becomes randomly available to a fraction $\lambda \in [0,1]$ of all dealers. We refer to those that obtain the information as being “informed”, and those that do not as “uninformed”. The identities of dealers who become informed is not known, and dealers are randomly matched as before.

This setting allows us to examine what happens to liquidity provision when the potential for adverse selection is increased. To ease the analysis, we assume that dealers must either choose $\delta \in (0, D - x)$ or exit the market. This preserves conditions that require participation to be incentive compatible, while requiring dealers to offer only executable spreads to traders. This allows us to abstract from dealers choosing fully-revealing market making.

A brief note on equilibrium selection. As is typical of asymmetric information models, multiple equilibria arise. In our setting, multiplicity arises from beliefs regarding inter-dealer trading strategies. To deal with this, we first characterize two classes of pure-strategy equilibria that together span the entire interval for $\lambda \in [0,1]$. For the subset of $\lambda$ for which both equilibria exist, we select the equilibrium that yields a greater ex-ante dealer profit whenever multiple equilibria exist.\(^{11}\) Importantly, the qualitative results outlined in this section do not depend on the exact selection method.

The key insight is that limited disclosure of information, by increasing asymmetries between dealers, may actually worsen liquidity relative to no disclosure. We illustrate this by considering how limited disclosure may affect the marginal value of inter-dealer trade of a dealer when all dealers have chosen some $\delta_i = \hat{\delta}$. In contrast to inter-dealer trading with no disclosure or full disclosure, dealers’ may sometimes reject trades that would offset their position.

For instance, an informed dealer may find it optimal to forgo trading with an opposite (uninformed) dealer because retaining his position may be more profitable than netting. Suppose that $\lambda$ is arbitrarily close to zero, and consider an informed long dealer’s optimal inter-dealer acceptance strategy. An uninformed short dealer expects the dealer she is matched with to also be uninformed and

\(^{11}\)Since all dealers are ex-ante identical, it chooses the equilibrium that is ex-ante desired by all dealers.
hence makes the offer \((\text{buy}, \bar{v} - \Delta)\) as specified in Lemma 1. Accepting this offer, rather than rejecting it, yields a difference in payoffs equal to \((\bar{v} - \Delta) - (v - \Delta)\), which is never positive when \(v = \bar{v} + x\). Likewise, consider the uninformed dealer’s offer strategy when \(\lambda\) is close to 1, i.e. when almost all dealers are informed. Since the receiver most likely knows \(v\), there is no point in following the strategy prescribed by Lemma 1. Rather, the uninformed dealer should offer either \(\bar{v} - x + \Delta\), in which case his offer is always accepted, or he should offer \(\bar{v} + x + \Delta\), which will only be accepted when \(v = \bar{v} + x\). When \(x > \Delta\) the latter strategy is preferred to the former.

We provide two lemmas that characterize the two classes of stage 2 equilibria of the inter-dealer game when \(x > 3\Delta\) and \(D > \frac{2x^2 + \Delta^2}{\Delta}\). The condition \(x > 3\Delta\) ensures that adverse selection, through the uncertainty regarding \(v\), is sufficiently important such that dealers strategically take it into consideration when trading in inter-dealer markets. In other words, it is a sufficient condition under which informed dealers forgo netting trades in favor of informationally driven trades, as we show in the case of Lemma 3. Similarly, it is a sufficient condition under which uninformed dealers choose prices that lower the likelihood of netting but protect themselves from being “cream-skimmed” by informed counterparties, as in the case of Lemma 4. The condition \(D > \frac{2x^2 + \Delta^2}{\Delta}\) is a sufficient condition for equilibrium existence, which we use later when characterizing the set of equilibria.

**Lemma 3** (Inter-dealer trading with pooling prices). Suppose that all dealers execute some partially-revealing offer at some spread \(\delta \in \left(0, \frac{Dx - x^2 + 2x\Delta}{x + \Delta}\right)\). For \(\lambda < \frac{2(D + x - \delta)\Delta}{2D(x - \Delta) + 3(D + x - \delta)\Delta}\), there exists a subgame equilibrium where in inter-dealer markets:

1. Uninformed short dealers make inter-dealer offer \((\sigma^*_s, P^d_s) = (\text{buy}, \bar{v} + \frac{\lambda}{\zeta - \lambda}x - \Delta)\), and only accept offers \((\text{sell}, P^d)\) for \(P^d = \bar{v} - \frac{\lambda}{\zeta - \lambda}x + \Delta\);

2. Uninformed long dealers make inter-dealer offer \((\sigma^*_l, P^d_l) = (\text{sell}, \bar{v} - \frac{\lambda}{\zeta - \lambda}x + \Delta)\), and only accept offers \((\text{buy}, P^d)\) for \(P^d = \bar{v} + \frac{\lambda}{\zeta - \lambda}x - \Delta\);

3. Informed dealers:

\[12\text{Note that as } \lambda \text{ approaches zero, almost all dealers are uninformed, in which case the trading strategies specified in Theorem 1 would be incentive compatible.}\]
• make inter-dealer offer \((\sigma^*_t, P^{d*}_t) = (sell, \bar{v} - \frac{\lambda}{2-\lambda} x + \Delta)\) if \(v = \bar{v} - x\), and only accept offers \((buy, P^d)\) for \(P^d = \bar{v} + \frac{\lambda}{2-\lambda} x - \Delta\);

• make inter-dealer offer \((\sigma^*_s, P^{d*}_s) = (buy, \bar{v} + \frac{\lambda}{2-\lambda} x - \Delta)\) if \(v = \bar{v} + x\), and only accept offers \((sell, P^d)\) for \(P^d = \bar{v} - \frac{\lambda}{2-\lambda} x + \Delta\);

4. Uninformed neutral dealers do not make any offers, and reject all offers.

**Lemma 4** (Inter-dealer trading with screening prices). Suppose that all dealers execute some partially-revealing offer at some spread \(\delta \in (0, D - x)\). For \(0 \leq \lambda \leq 1\), there exists a subgame equilibrium where in inter-dealer markets:

1. Informed short dealers make inter-dealer offer \((\sigma^*_s, P^{d*}_s) = (buy, \bar{v} - \Delta), and only accept offers \((sell, P^d)\) for \(P^d = \bar{v} - \Delta\);

2. Uninformed short dealers make inter-dealer offer \((\sigma^*_s, P^{d*}_s) = (buy, \bar{v} - x - \Delta), and only accept offers \((sell, P^d)\) for \(P^d = \bar{v} + x - \Delta\);

3. Informed long dealers make inter-dealer offer \((\sigma^*_l, P^{d*}_l) = (sell, \bar{v} + \Delta), and only accept offers \((buy, P^d)\) for \(P^d = \bar{v} + \Delta\);

4. Uninformed long dealers make inter-dealer offer \((\sigma^*_l, P^{d*}_l) = (sell, \bar{v} + x + \Delta), and only accept offers \((buy, P^d)\) for \(P^d = \bar{v} - x + \Delta\);

5. Neutral dealers do not make any offers, and reject all offers.

In the first inter-dealer equilibrium characterized in Lemma 3, dealers make offers with “pooling” prices, which are accepted by uninformed dealers. In this equilibrium, offering dealers must offer increasingly attractive prices for greater \(\lambda\), as uninformed receiving dealers face increasing adverse selection when the possibility that the trade is being initiated by an informed counterparty is greater. In this equilibrium, informed dealers ignore netting benefits and trade primarily based on information. When \(x\) becomes sufficiently large relative to \(\Delta\), an informed dealer forgoes trades that would offer mutually beneficial netting, and instead accepts trades based on his informational advantage. This implies that an informed dealer may increase his net position from 1 to 2 if long or short, and 0 to 1 if neutral, if information warrants it. Importantly, informed neutral dealers, who did not trade in inter-dealer markets in Theorems 1 and 3, actively trade in
the inter-dealer market to extract purely informational rents from less-informed dealers.

In the second inter-dealer equilibrium characterized in Lemma 4, uninformed dealers instead use “screening” prices, which are only sometimes accepted by informed dealers. At the cost of failing to net out their position, uninformed dealers insulate themselves from a winner’s curse problem, which they would face under a pooling price equilibrium as in Lemma 3. In general, the benefit of screening prices increases with $\lambda$, as the likelihood of meeting an informed counterparty increases. Informed dealers offer $v \pm \Delta$, which are always accepted by an informed dealer of an opposite position, or an uninformed dealer when the price is attractive (e.g. an offer to buy at a high price).

Given these two equilibrium characterizations we can define equilibria with inter-dealer trading for any amount of partial information disclosure. The characterization depends on the existence of a unique cut-off $\bar{\lambda}$ at which dealers in stage 1 pivot from the $\delta^*$ strategy to the $\delta^{**}$ strategy. Intuitively, the change in the equilibrium strategies employed by dealers over the span of $\lambda$ reflect the extent to which counterparties are likely to be informed. When uninformed, a dealer trades off the benefits of offsetting his position at the adverse selection discount he must offer to ensure trade will be accepted by an uninformed counterparty against the benefits of offering a high price to avoid being “scalped” by an informed counterparty. When informed, a dealer trades off the gains from acting on his information strategically against the gains from maximizing trade with an informed counterparty.

**Theorem 4** (Limited Disclosure). Suppose that $x > 3\Delta$ and $D > \frac{2x^2 + \Delta^2}{\Delta}$. There exists $\bar{\lambda}$ with $0 < \lambda < 1$ such that:

- For $\lambda \leq \bar{\lambda}$ there exists an equilibrium with inter-dealer trading in all dealers make bid-ask offers $(\bar{v} - \delta^*, \bar{v} + \delta^*)$ where $\delta^* = \frac{2D^2 + (1-\lambda)x^2 + (1+\lambda)\Delta D + (1-\lambda)\lambda(x-\Delta)(D-x)}{4D(1-\lambda)\Delta}$ and traders sell if and only if $v_j \leq \bar{v} - \delta^*$ and buy if and only if $v_j \geq \bar{v} + \delta^*$.

- For $\lambda > \bar{\lambda}$ there exists an equilibrium with inter-dealer trading in all dealers make bid-ask offers $(\bar{v} - \delta^{**}, \bar{v} + \delta^{**})$ where $\delta^{**} = \frac{2D^2 + (2-\lambda)\Delta D}{4D(\Delta)}$ and traders sell if and only if $v_j \leq \bar{v} - \delta^{**}$ and buy if and only if $v_j \geq \bar{v} + \delta^{**}$.

Theorem 4 states that when a small set of dealers become informed, i.e. $\lambda < \bar{\lambda}$, uninformed dealers offer discounted offers to attract uninformed counterparties,
(a) Dealers’ Expected Payoffs over $\lambda$.  

(b) Liquidity provision as a function of $\lambda$.

Figure 4: This figure provides an example in which $D = 29$, $x = 3$, and $\Delta = 1$.  

who face adverse selection, in order to maximize the likelihood of trade. Informed dealers take advantage of discounted offers by choosing to trade only when the fundamentals $v$ are in their favor. However, when a large set of dealers become informed, i.e. $\lambda > \bar{\lambda}$, uninformed dealers choose to make defensive offers, as they are likely to match with informed counterparties. Informed dealers, instead, offer a price that reflects the fundamental value $v$ in order to successfully trade with informed counterparties.

Figure 4 shows an example of the equilibrium payoffs and liquidity provision $\mu$ over $\lambda \in [0, 1]$. The solid curve represents the equilibrium, for which at a critical value of $\lambda$ equilibrium inter-dealer trading switches from pooling to screening prices. The “shadow” equilibrium outcome under screening prices is depicted in the dotted curve.

An important consideration is the extent to which limited disclosure of information affects market liquidity. Given that the spread $\delta$ captures the amount of liquidity offered by dealers, we can characterize market liquidity over the interval $\lambda \in [0, 1]$.

\textsuperscript{13}In this example, the cutoff specified in Lemma 3 is binding. As such, the equilibrium switches to a screening price equilibrium even though pooling prices would be ex-ante more profitable. Correspondingly, the expected profits are discontinuous at $\bar{\lambda}$.  

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Corollary 3 (Limited Disclosure and Liquidity). Suppose that $x > 3\Delta$ and $D > \frac{2\Delta^2 + \Delta^2}{\Delta}$. Then, liquidity takes a disjointed V-shape over the interval $\lambda \in (0, 1)$.

Corollary 3 states that while equilibrium liquidity is greater under full disclosure of post-trade information than under no disclosure, limited disclosure may be inferior to both. The disjointed V-shape pattern, shown in Figure 4b reflects competing forces that impact equilibrium market liquidity. First, there is an unambiguously positive effect: for any $\lambda > 0$ translates into an injection of information into the system. Dealers, expecting that they will be better informed in the inter-dealer market, can offer more competitive market-making. Second, there is an unambiguously negative effect: for any $\lambda < 1$, dealers not only have divergent beliefs, but are also asymmetrically informed. This sprouts an adverse selection problem that decreases the set of successful trades in inter-dealer markets. When the gains from strategically trading as an informed dealer is sufficiently high, the negative force initially overpowers the first, until sufficiently many dealers are likely to become informed.

7 Strategic Platform

In the previous section we saw that liquidity provision under limited disclosure of information can be worse than under either full or no disclosure. This raises the question as to why asymmetric information structures would exist in practice. Here we suggest an answer that follows from the fact that in environments where a platform controls access to post trade information, that platform may have incentives to offer access to information in the worst possible way. This is because the return to the sale of post trade information is highest when the information is most valuable, a situation that occurs when the adverse selection costs to not be informed are the highest.

Suppose there exists a strategic platform that has information regarding all stage 1 trades. At the beginning of time $t = 2$, the platform chooses a cost $c_\theta$ at which any dealer of type $\theta$ can observe this information. Thus, a dealer of type $\theta$ that chooses to become informed pays cost $c_\theta$ to the platform, while a dealer that does not remains “uninformed”. We allow for dealers to independently choose
whether to become informed or not.\textsuperscript{14} Furthermore, we assume that the platform cannot commit to any cost schedule $c_\theta$ at $t = 1$, and dealers cannot commit to any acquisition strategies. For expositional reasons, we restrict our attention to cost schedules $c_\theta$ such that the fraction of informed dealers by type is identical, as in Section 6.\textsuperscript{15}

Suppose dealers share beliefs that other dealers will follow inter-dealer trading strategies that maximize stage 2 payoffs given some $\delta$. This means that inter-dealer trading follows the strategies specified in Lemma 3 if and only if $\delta \in \left(0, \frac{Dx-x^2+2x\Delta}{x+2\Delta}\right)$ and $\lambda < \frac{2(D+x-\delta)\Delta}{2D(x-\Delta)+3(D+x-\delta)\Delta}$. Under the same conditions we considered in Section 6, the strategic platform endogenously chooses a cost schedule that implements limited disclosure of information.

**Theorem 5.** Suppose that $x > 3\Delta$ and $D > \frac{2x^2+\Delta^2}{\Delta}$. In equilibrium, the strategic platform strictly prefers limited disclosure of information to full disclosure. Specifically, the strategic platform optimally chooses a cost schedule $c_\theta^\circ$ that induces $\lambda^\circ < 1$.

The strategic platform strictly prefers limited disclosure of information, and sets the cost $c_\theta$ to be so high that only a subset of dealers choose to become informed. Crucially, the platform maximizes its profits by setting the cost of information for each dealer type equal to the informational rent each dealer type obtains from becoming informed. Under Lemma 3, information becomes more valuable as $\lambda$ increases. Hence, the platform optimally chooses a cost schedule $c_\theta^\circ$ that induces the maximum $\lambda$ under which inter-dealer trading follows that in Lemma 3. In addition, dealers’ optimal bid-ask spread given $\lambda$, $\delta(\lambda) = \frac{2D^2+(1-\lambda)x^2+\Delta D+\lambda x D}{4D-(1-\lambda)\Delta}$, increases in $\lambda$. This implies the following result:

**Corollary 4.** Dealers’ liquidity provision in the equilibrium described in Theorem 5 is the lowest possible under Lemma 3.

## 8 Conclusion

In this paper, we develop a model of decentralized asset markets with a tiered trading structure to study market liquidity in a setting in which dealers face both

\textsuperscript{14}In other words, we do not impose symmetry with respect to acquisition strategy.

\textsuperscript{15}This allows a comparison between the outcome with a strategic platform to the exogenous case outlined in Theorem 6, as inter-dealer trading will follow either pooling prices or separating prices, as in Lemmas 3 and 4. It can be shown that the qualitative results hold more generally.
adverse selection and liquidity costs. We show that inter-dealer trading endogenously arises when the benefits of liquidity management outweigh adverse selection costs, and, in doing so, we demonstrate how market liquidity is tightly linked to inter-dealer liquidity. When adverse selection is too severe, inter-dealer trading ceases to exist and markets become segmented. We build on this framework to study how the disclosure of information impacts market liquidity. Full disclosure of information on trades in the market-making stage increases overall liquidity relative to no disclosure, but the transition from zero to full disclosure is not monotonic. Informing only a subset of dealers initially leads to less liquidity than the no disclosure case as it creates problems of asymmetric information. As more and more dealers are informed these problems become small relative to the advantages of being close to a state where everyone is informed. Interestingly, we show that the worst possible case of limited disclosure emerges as an equilibrium outcome in an environment where post trade information is sold by a strategic platform. This result supports regulations that discourage or limit premium subscriptions to trading information; see, for example, recent statements by the Securities and Exchange Commission that put greater scrutiny on price increases of market data by the Nasdaq and New York Stock Exchange.\textsuperscript{16}

Full disclosure of post-trade information maximizes liquidity provision in our model, but it does not eliminate all of the frictions that potentially limit trade. Two other frictions persist in our description of a tiered OTC market that prevent the market solution from maximizing aggregate social welfare. The first arises because of a positive externality: individual dealers do not internalize the benefit that arises to other dealers, through increased trading opportunities in the inter-dealer market, when they lower their spread. The second friction arises because our model gives market power to the dealers in the market-making stage. The first friction can be eliminated by solving the planners problem in which trader welfare is maximized. Both frictions can be eliminated by maximizing trader welfare subject to a participation constraint for the dealers.\textsuperscript{17}

\textsuperscript{16}Specifically, in October 2018, the SEC announced that these exchanges had not provided sufficient evidence that fees were “fair and reasonable and not unreasonably discriminatory” (see https://www.sec.gov/news/public-statement/statement-chairman-clayton-2018-10-16), and in May 2019, published staff guidance regarding fee filings (see https://www.sec.gov/tm/staff-guidance-sro-rule-filings-fees).

\textsuperscript{17}These exercises are conducted in Appendix B.
remaining frictions not only increases liquidity provision in equilibrium, but also increases the parameter set for which equilibria with inter-dealer trading exist.

References


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A Proofs

A.1 Proof of Lemma 1

We characterize the inter-dealer market at \( t = 2 \), taking as given some bid-ask spreads \((P_b, P_a)\) corresponding to spread \( \delta < D - x \) used by dealers in the market making stage at \( t = 1 \). In the inter-dealer market, three potential types of dealers arise – short dealers, long dealers, and neutral dealers.

We guess and verify that the inter-dealer equilibrium strategies of dealers are such that:

- short dealers make offer (buy, \( P^d_s \)) only accepted by long dealers;
- long dealers make offer (sell, \( P^d_l \)) only accepted by short dealers;
- neutral dealers chooses not to trade, i.e. \( \sigma_n = \text{no trade} \).

Correspondingly, let beliefs be such that any buy offer (i.e. \( \sigma = \text{buy} \)) is made by a short dealer and any sell offer (i.e. \( \sigma = \text{sell} \)) by a long dealer. The beliefs of a pair of dealers are identical conditional on successfully trading, since

\[
P(\bar{v} = \bar{v} - x | \text{short dealer matches with long dealer}) = \frac{(P_b - \bar{v} + x + D)(\bar{v} - x + D - P_a)}{(P_b - \bar{v} + x + D)(\bar{v} - x + D - P_a) + (P_b - \bar{v} - x + D)(\bar{v} + x + D - P_a)} \]
\[
= \frac{1}{2} = P(\bar{v} = \bar{v} + x | \text{short dealer matches with long dealer}) \tag{7}
\]

Given these beliefs, a short dealer (long dealer) makes an offer that are equal to the reservation price of a long dealer (short dealer), which is equal to \( \bar{v} - \Delta \) (\( \bar{v} + \Delta \)). We must verify that deviations are not profitable. There are three classes of deviations: (1) neutral dealers accepting equilibrium offers from other dealers, (2) neutral dealers making offers to other dealers and (3) long or short dealers making offers at prices other than the proposed equilibrium prices.
Step 1. Show that neutral dealers have no incentive to deviate by accepting an equilibrium offer from the dealer they are matched with in stage 2.

Part 1A. Suppose a neutral dealer receives an offer to buy at price \( \bar{v} + \Delta \) from a dealer she is matched with in stage 2. Given our specified equilibrium strategies, she believes the offer is coming from a long dealer and hence her expectation of \( v \), the true value, becomes \( E_l[v] = \bar{v} - \frac{x^2}{D-\delta} < \bar{v} \). If she accepted the offer to buy, her expected payoff from accepting the offer would be \( \bar{v} - \frac{x^2}{D-\delta} - \Delta - [\bar{v} + \Delta] = -[\frac{x^2}{D-\delta} + 2\Delta] < 0 \). So the neutral dealer will not accept an offer to buy at the price \( \bar{v} + \Delta \).

Part 1B. Suppose a neutral dealer receives an offer to sell at price \( \bar{v} - \Delta \) from a dealer she is matched with in stage 2. Given our specified equilibrium strategies, she believes the offer is coming from a short dealer and hence her expectation of \( v \), the true value, becomes \( E_s[v] = \bar{v} + \frac{2x^2}{2(D-\delta)} > \bar{v} \). If she accepted the offer to sell, her expected payoff from accepting the offer would be \( \bar{v} - \Delta - [\bar{v} + \frac{2x^2}{2(D-\delta)} + \Delta] = -[\frac{x^2}{D-\delta} + 2\Delta] < 0 \). So the neutral dealer will not accept an offer to sell at the price \( \bar{v} - \Delta \).

Step 2. Show that no neutral dealer has an incentive to make a buy or sell offer to a dealer they are matched with in stage 2.

A neutral dealer has no incentive to make an offer to a counterparty. To see this, consider a deviation in which a neutral dealer makes an offer to sell. First consider when a neutral dealer offers to sell at \( P' = \bar{v} + \Delta \). Given equilibrium beliefs, the offer is accepted if the neutral dealer is matched to a short dealer, which yields the following payoff

\[
P' - E_n[v]\text{neutral dealer sells to short dealer] - \Delta = \bar{v} - E_n[v]\text{neutral dealer sells to short dealer] < 0}
\]

Since conditional on matching with a short dealer, the conditional expected value of \( v \) is greater than \( \bar{v} \), the deviation is not profitable. Second, consider when a neutral dealer offers some \( P' < \bar{v} + \Delta \), which is only potentially accepted by a short dealer. Given off-equilibrium beliefs, a short dealer’s valuation of the asset conditional on receiving an offer from the neutral dealer is equal to \( \bar{v} \). Hence, there does not exist any \( P' \) such that the neutral dealer makes a positive profit from making an offer. A symmetric argument holds for deviations by the neutral dealer to make a buying offer.

Step 3. Show that no long or short dealer would want to deviate by making an offer at a price different than the proposed equilibrium price.

Part 3A. Consider a long dealer (who in equilibrium offers to sell at a price \( \bar{v} + \Delta \) that is accepted only by short dealer). Recall that we assume dealers have beliefs that are triggered by any sell offer, not just an offer at the equilibrium price,
and these beliefs are that the dealer making the sell offer is a long dealer. So a deviation to a price $P' > \bar{x} + \Delta$ would be rejected by a short dealer: it would be deemed unprofitable given updated beliefs that the asset’s true value is $\bar{x}$. And it would be rejected by a neutral dealer who would have even more pessimistic beliefs about the asset value. What about a price $P'' < \bar{x} + \Delta$? By offering a price less than $\bar{x} + \Delta$ the long dealer would be giving up some surplus when matched with a short dealer. The question is whether she can recoup that when matched with a neutral dealer. However, in order to get a neutral dealer to accept a sell offer she must offer a price of $E_i[v] - \Delta = \bar{x} - \frac{x^2}{D-y} - \Delta$, but this is exactly the long dealer’s expected payoff if she does not sell the asset in stage 2. So, by offering a price less than $\bar{x} + \Delta$ the long dealer loses surplus when matched with a short dealer and makes no additional surplus when matched with a neutral dealer. So this deviation is not profitable.

Part 3B. Consider a short dealer (who in equilibrium offers to buy at a price $\bar{x} - \Delta$ that is accepted only by long dealer). A similar argument to part 3A shows that no deviation in price is profitable.

A.2 Proof of Theorem 1

The equilibrium is verified by solving backwards. We proceed in three steps. First, we characterize the optimal inter-dealer trading strategy of a dealer $i$ who chose some market-making strategy $\delta_i$ at $t = 1$, taking as given that all other dealers choose some market-making strategy $\hat{\delta} \in (0, D - x)$ at $t = 1$ and follow inter-dealer trading strategies specified in Lemma 1. Second, by backward induction, we characterize the expected payoff at $t = 1$ of an individual dealer’s who chooses the market-making strategy $\delta_i$ taking as given that all other dealers choose $\hat{\delta}$. Third, we determine the conditions under which there exists some $\delta^* \in (0, D - x)$ such that an individual dealer maximizes his expected payoff by choosing $\delta_i = \delta^*$ conditional on all other dealers choosing $\delta^*$.

Step 1. We begin by characterizing dealers’ strategies in the inter-dealer market at $t = 2$. Following Lemma 1, consider the following set of candidate equilibrium strategies:

1. short dealers make offer ($buy, \bar{x} - \Delta$) and only accept offers ($sell, P^d$) for $P^d \leq \bar{x} + \Delta$;
2. long dealers make offer ($sell, \bar{x} + \Delta$) and only accept offers ($buy, P^d$) for $P^d \geq \bar{x} - \Delta$;
3. neutral dealers do not make any offers, and reject all offers.

Correspondingly, let dealers’ beliefs be such that any buy offer (i.e. $\sigma = buy$) is made by a short dealer and any sell offer (i.e. $\sigma = sell$) by a long dealer. We
must verify that given that all other dealers choose some market-making strategy \( \hat{\delta} \in (0, D - x) \), an individual dealer \( i \) does not find it profitable to deviate to \( \delta_i \neq \hat{\delta} \).

First, consider when a dealer selects some \( \delta_i > \hat{\delta} \), so that the dealer is better informed than other dealers. Given dealer beliefs, dealer \( i \)'s optimal inter-dealer offer and trading strategy is to mimic other dealers’ equilibrium strategy. As an offering dealer, dealer \( i \) maximizes conditional profits by offering \( (\text{sell}, \bar{v} + \Delta) \) and \( (\text{buy}, \bar{v} - \Delta) \), as these are the maximum and minimum prices at which a receiving dealer is willing to buy or sell, respectively. Similarly, a receiving dealer \( i \)'s optimal inter-dealer trading strategy is to mimic other the dealers’ equilibrium strategy and accept \( (\text{sell}, \bar{v} + \Delta) \) as a short dealer and accept \( (\text{buy}, \bar{v} - \Delta) \) as a long dealer. Given this, the marginal payoff from inter-dealer trading \( V_\theta(\delta_i, \hat{\delta}) \) of a long or short dealer \( i \) at \( t = 2 \) is given by

\[
V_\theta(\delta_i, \hat{\delta}) = \left( \sum_v P(v|\theta) P(\text{match with opposite dealer}|v, \theta) \right) (\bar{v} - E_i[v|\text{trade}]) + \left( \sum_v P(v|\theta) P(\text{match with opposite dealer}|v, \theta) \right) \Delta.
\]  

for \( \theta \in \{l, s\} \). Here, the probability that dealer \( i \) matches with an opposite dealer conditional on becoming a long (or short) dealer is given by

\[
\sum_v P(v|\theta) P(\text{match with opposite dealer}|v, \theta) = \begin{cases} 
\frac{(D - \delta_i)(D - \hat{\delta}) - x^2}{2D(D - \hat{\delta})} & \text{if } \delta_i \leq D - x \\
\frac{D - x - \delta}{2D} & \text{if } \delta_i > D - x.
\end{cases}
\]  

(10)

We can explicitly characterize the terminal payoff in the first component of (9):

\[
\bar{v} - E[v|\text{trade}] = \begin{cases} 
\frac{(\delta_i - \bar{v})x}{(D - \delta_i)(D - \hat{\delta}) - x^2} x & \text{if } \delta_i \leq D - x \\
x & \text{if } \delta_i > D - x.
\end{cases}
\]  

(11)

The remaining case is when a dealer selects some \( \delta_i < \hat{\delta} \), such that the dealer is less informed than other dealers. The optimal inter-dealer trading strategy involves rejecting any equilibrium offer, since \( \delta_i < \hat{\delta} \) implies that upon obtaining a short position the dealer has a lower conditional expected value of \( v \) than a dealer that chose \( \hat{\delta} \), and upon obtaining a long position the dealer has a higher conditional expected value of \( v \) than a dealer that chose \( \hat{\delta} \). Second, mimicking
the equilibrium offer strategy is more profitable than no trade if the gains from netting outweigh the losses associated with negative information rents. This holds true if the following inequality holds:

$$\Delta > -\frac{(\delta_i - \hat{\delta}) x}{D^2 - x^2 + \delta_i \hat{\delta} - D(\delta_i + \hat{\delta})}.$$  \hspace{1cm} (12)

Now, we can express $V_\theta(\delta_i, \hat{\delta})$ conditional on whether (12) holds or not. If (12) does not hold, then the dealer rejects all offers and does not make any offer. Hence, $V_\theta(\delta_i, \hat{\delta}) = 0$. Together, the marginal inter-dealer payoff is given by:

$$V_\theta(\delta_i, \hat{\delta}) = \begin{cases} \max \left\{ 0, \frac{(D - \delta_i)(D - \hat{\delta}) - x^2}{2D(D - \delta_i)} \left( \frac{\delta_i - \hat{\delta}}{D^2 - x^2 + \delta_i \hat{\delta} - D(\delta_i + \hat{\delta})} x + \Delta \right) \right\} & \text{if } \delta_i \leq \hat{\delta} \\
\frac{(D - \delta_i)(D - \hat{\delta}) - x^2}{2D(D - \delta_i)} \left( \frac{\delta_i - \hat{\delta}}{D^2 - x^2 + \delta_i \hat{\delta} - D(\delta_i + \hat{\delta})} x + \Delta \right) & \text{if } \delta_i \in (\hat{\delta}, D - x) \\
\frac{D - x - \hat{\delta}}{2D} (x + \Delta) & \text{if } \delta_i > D - x. \end{cases}$$  \hspace{1cm} (13)

**Step 2.** Now that we have fully characterized a dealer $i$’s optimal inter-dealer strategy conditional on deviating to $\delta_i \neq \hat{\delta}$, we can backward induct, and fully characterize the expected payoff at $t = 1$ conditional on deviating. The generic $t = 1$ expected payoff of dealer $i$ is given by

$$\Pi_i(\delta_i, \hat{\delta}) = P(\gamma_j(P^b, P^a) = \text{accept} | \delta_i) \cdot (\bar{\theta} + \delta_i - E[v | \hat{\delta}_i] - \Delta) + \sum_\theta P(\theta_i = \theta | \hat{\delta}_i) \cdot V_\theta(\delta_i, \hat{\delta}).$$  \hspace{1cm} (14)

Since the market-making payoff $A$ is independent of $\hat{\delta}$ or inter-dealer payoffs, we can express this conditional on whether $\delta_i \leq D - x$ or $\delta_i > D - x$:

$$A = \begin{cases} 2 \cdot \frac{D - \delta_i}{2D} \left( \delta_i - \frac{x}{D - \delta_i} \cdot x - \Delta \right) & \text{if } \delta_i \leq D - x \\
2 \cdot \frac{D + x - \delta_i}{4D} (\delta_i - x - \Delta) & \text{if } \delta_i > D - x. \end{cases}$$  \hspace{1cm} (15)

As we did in the previous step, we split the analysis into the cases in which $\delta_i > \hat{\delta}$ and $\delta_i < \hat{\delta}$. First, suppose $\delta_i > \hat{\delta}$. Incorporating our earlier expression of $V_\theta(\delta_i, \hat{\delta})$, the inter-dealer payoff $B$ is given by

$$B = \begin{cases} 2 \cdot \frac{D - \delta_i}{2D} \left( \frac{\delta_i - \hat{\delta}}{D(D - \delta_i)} x + \frac{D^2 - x^2 + \delta_i \hat{\delta} - D(\delta_i + \hat{\delta})}{2D(D - \delta_i)} \Delta \right) & \text{if } \delta_i \leq D - x \\
\frac{D + x - \delta_i}{2D} \frac{D + x - \hat{\delta}}{2D} (x + \Delta) & \text{if } \delta_i > D - x. \end{cases}$$  \hspace{1cm} (16)
Together, we can express the expected payoff at $t = 1$ for dealer $i$ given some market-making strategy $\delta_i > \hat{\delta}$:

$$
\Pi_i(\delta_i, \hat{\delta}) = \begin{cases} 
\frac{D-\delta_i}{D} \left( \delta_i \frac{x}{D-\delta_i} \cdot x - \Delta \right) + \frac{D-\delta_i}{D} \left( \frac{(\delta_i-\hat{\delta})x}{2D(D-\delta_i)} \cdot x + \frac{D^2 - x^2 + \delta_i \hat{\delta} - D(\delta_i + \hat{\delta})}{2D(D-\delta_i)} \Delta \right) & \text{if } \delta_i \leq D - x \\
\frac{D+x-\delta_i}{2D} \left( \delta_i \frac{x}{D-\delta_i} \cdot x - \Delta \right) + \frac{D+x-\delta_i}{2D} \frac{D-x-\hat{\delta}}{2D} (x + \Delta) & \text{if } \delta_i > D - x.
\end{cases}
$$

(17)

Second, suppose $\delta_i < \hat{\delta}$. Incorporating our earlier expression of $V_\theta(\delta_i, \hat{\delta})$, the inter-dealer payoff $B$ is given by

$$
B = \max \left\{ 2 \cdot \frac{D - \delta_i}{2D} \left( \frac{(\delta_i - \hat{\delta})x}{2D(D-\delta_i)} \cdot x + \frac{D^2 - x^2 + \delta_i \hat{\delta} - D(\delta_i + \hat{\delta})}{2D(D-\delta_i)} \Delta \right), 0 \right\}.
$$

(18)

Together, we can express the expected payoff at $t = 1$ for dealer $i$ given some market-making strategy $\delta_i < \hat{\delta}$:

$$
\Pi_i(\delta_i, \hat{\delta}) = \frac{D - \delta_i}{D} \left( \delta_i \frac{x}{D - \delta_i} \cdot x - \Delta \right) + \max \left\{ 2 \cdot \frac{D - \delta_i}{2D} \left( \frac{(\delta_i - \hat{\delta})x}{2D(D - \delta_i)} \cdot x + \frac{D^2 - x^2 + \delta_i \hat{\delta} - D(\delta_i + \hat{\delta})}{2D(D - \delta_i)} \Delta \right), 0 \right\}.
$$

(19)

**Step 3.** Given that we have a characterization of a dealer $i$’s expected payoff from choosing $\delta_i$, it suffices to determine the conditions under which a symmetric equilibrium with inter-dealer markets exist, and correspondingly dealers’ equilibrium strategies.

We do so by first identifying the local optimal $\delta_i \in (\hat{\delta}, D - x)$. Taking the first order condition of (17) with respect to $\delta_i$ and rearranging terms yields

$$
\frac{2D^2 + x^2 + \Delta D}{4D} + \frac{\Delta}{4D} \hat{\delta} = \delta_i.
$$

(20)

Imposing symmetry by setting $\hat{\delta} = \delta_i$, we obtain

$$
\delta^* = \frac{2D^2 + x^2 + \Delta D}{4D - \Delta}.
$$

(21)

In order for (21) to be an equilibrium solution, the expected payoff from $\delta^*$ must be greater than deviating to any $\delta_i \in (0, D + x)$. Consider any $\delta_i \in (0, \delta^*)$. First, note that for any $\delta_i$ such that (12) holds, $\delta^*$ is the optimum, since in this case, payoff function over $\delta_i \in (0, D - x)$ is continuous and differentiable. Second, note
that if $\frac{\partial B}{\partial \delta_i} < 0$, then $\delta^* > \frac{D+\Delta}{2}$, which implies that any $\delta_i < \delta^*$ yields a lower expected payoff. This leaves the cases in which $\frac{\partial B}{\partial \delta_i} > 0$ and (12) is violated. Note that $\frac{\partial B}{\partial \delta_i} > 0$ if and only if $2x^2 > \Delta(D - \Delta)$. This condition holds as long as $x > \sqrt{\frac{\Delta(D-\Delta)}{2}}$. Suppose that it holds. In this case, a dealer $i$'s payoff is

$$\frac{D - \delta_i}{D} (\delta_i - \Delta) - \frac{x^2}{D} < \frac{D - \delta_i}{D} (\delta_i - \Delta) - \frac{\Delta(D - \Delta)}{2D}$$

For any $\Delta \in \left(\frac{D}{\sqrt{2}+1}, D\right)$, the above is less than 0. Hence, there does not exist any profitable deviation to some $\delta_i < \delta^*$. Next, consider deviations to some $\delta_i \in (D - x, D + x)$. Recall, the expected payoff from choosing $\delta_i \in (D - x, D + x)$, given by (17) is

$$\frac{D + x - \hat{\delta}_i}{2D} (\delta_i - x - \Delta) + \frac{D + x - \hat{\delta}_i}{2D} \frac{D - x - \hat{\delta}_i}{2D} (x + \Delta). \quad (22)$$

Note that as $x \to 0$, the probability of successfully trading in the market making stage approaches 0, for any $\delta' \in (D - x, D + x)$. As $x \to 0$, for any partially revealing market-making strategy $\delta'$, profits approach

$$\frac{(D - \delta_i)\delta_i}{D} - \Delta \cdot \frac{(D + \delta^*)(D - \delta_i)}{2D^2} > 0. \quad (23)$$

This implies that there exists some cutoff value $x^{\text{trade}}$ such that for $x < x^{\text{trade}}$, deviation to $\delta_i > D - x$ is not profitable.

### A.3 Proof of Theorem 2

As established in Section (4.2), if dealers collectively use fully revealing market-making strategies, then inter-dealer trading does not occur. To show that all dealers playing fully revealing market-making strategies can comprise an equilibrium suppose that all dealers other than dealer $i$ choose $\hat{\delta} \in (D - x, D + x)$ and consider dealer $i$'s payoff from choosing some arbitrary $\delta_i$. We begin by considering $\delta_i \in (D - x, D + x)$. Since these strategies are full revealing, no other dealer will trade in the inter-dealer market and hence the payoff to dealer $i$ is given by the market-making payoff

$$\Pi(\delta_i, \hat{\delta} | \delta_i \geq D - x) = \frac{D + x - \delta_i}{2D} (\delta_i - x - \Delta) \cdot \overset{\text{market-making payoff}}{\text{(24)}}$$
Notably, given $\delta_i \in (D - x, D + x)$, a dealer’s payoff is independent of $\hat{\delta}$, since he does not expect to trade with any other dealer in the inter-dealer stage. Hence, dealer $i$’s best response $\delta_i$ simply maximizes his market-making payoff

$$\delta_i(\hat{\delta}) = x + \frac{D + \Delta}{2}. \quad (25)$$

$x + \frac{D + \Delta}{2} > D - x$ if $x > \frac{D - \Delta}{2}$. It remains to show that dealer $i$ will not want to choose some $\delta_i < D - x$ given $\hat{\delta} \in (D - x, D + x)$. Given the expectation of no inter-dealer trading, the payoff is still the market-making payoff, which is now given as

$$\Pi(\delta_i, \hat{\delta} | \delta_i < D - x) = \frac{D - \delta_i}{D} \left( \delta_i - \frac{x^2}{D - \delta_i - \Delta} \right). \quad (26)$$

Dealer $i$ maximizes his payoff for $\delta_i = \frac{D + \Delta}{2}$. Note that fully revealing $\delta_i$ yields a greater payoff if and only if

$$\frac{D + x - \left( x + \frac{D + \Delta}{2} \right) \left( x + \frac{D + \Delta}{2} - x - \Delta \right)}{2D} = \frac{1}{2D} \left( \frac{D - \Delta}{2} \right)^2 > \frac{1}{D} \left( \frac{D - \Delta}{2} \right)^2 - \frac{x^2}{D}. \quad (27)$$

This condition is satisfied if $x > \frac{D - \Delta}{2\sqrt{2}} = x_{\text{notrade}}$. This pins down the conditions under which an equilibrium without inter-dealer trading exists, where $\delta^{**} = x + \frac{D + \Delta}{2}$.

### A.4 Proof of Theorem 3

The proof follows the proof of Theorem 1. The key departure is that all dealers know $v$ prior to trading in stage 2. We guess and verify that the inter-dealer equilibrium strategy of the dealers are such that:

1. short dealers make the offer $(\text{buy}, P_s^d(v))$, which is only accepted by long dealers
2. long dealers make the offer $(\text{sell}, P_l^d(v))$, which is only accepted by short dealers
3. neutral dealers choose not to trade, i.e., $\sigma_n = \text{no trade}$.

Note that prices $P_s^d, P_l^d$ are conditional on $v$ to reflect that it is publicly observable. Hence, the reservation price at which a receiving dealer is willing to buy is $v -$
\(\Delta, v + \Delta, v - \Delta\) for long, short, and neutral dealers, respectively, and willing to sell is \(v - \Delta, v + \Delta, v + \Delta\) for long, short, and neutral dealers, respectively. It directly follows that no information rents can be extracted through inter-dealer trading, i.e. the marginal payoff from inter-dealer trading \(V_\theta(\delta_i, \hat{\delta})\) of dealer \(i\) at \(t = 2\) is given by

\[
V_\theta(\delta_i, \hat{\delta}) = (P(v|\theta) P(\text{match with opposite dealer}|v, \theta)) \Delta. \tag{28}
\]

The conjectured inter-dealer strategies are incentive compatible, since any trades between dealer matches other than short to long yield zero gains to either dealer (and positive for long-short). In addition, \(P_s^d = v - \Delta\) and \(P_f^l = v + \Delta\). As before, we solve for equilibrium market-making strategy (denoted \(\delta^{***}\)) via backward induction. The expected payoff at \(t = 1\) for dealer \(i\) given some market-making strategy \(\delta_i\) is

\[
\Pi_i(\delta_i, \hat{\delta}) = \begin{cases} 
\frac{D - \delta_i}{D} \left( \delta_i - \frac{x}{D - \delta_i} \cdot x - \Delta \right) + \frac{D - \delta_i}{D} \left( \frac{D^2 - x^2 + \delta_i \delta_i - D(\delta_i + \hat{\delta})}{2D(D - \delta_i)} \right) & \text{if } \delta_i \leq D - x \\
\frac{D + x - \delta_i}{2D} \left( \delta_i - x - \Delta \right) + \frac{D + x - \delta_i}{2D} \frac{D - x - \hat{\delta}}{2D} & \text{if } \delta_i > D - x
\end{cases}
\tag{29}
\]

The first order necessary condition for optimal \(\delta_i \leq D - x\) is

\[2D^2 + D\Delta = 4D\delta_i - \hat{\delta}\Delta. \tag{30}\]

Applying symmetry yields \(\delta^{***} = \frac{2D^2 + D\Delta}{4D - \Delta}\).

In order for \(\delta^{***}\) to be an equilibrium solution, the expected payoff from \(\delta^{***}\) must be greater than deviating to any \(\delta_i > D - x\). We show that there exists some threshold \(x^{\text{trade, disclosure}}\) such that this is true for \(x < x^{\text{trade, disclosure}}\). First, note that as \(x \to D\), it trivially holds that an individual dealer strictly prefers to deviate to some \(\delta_i > D - x\). This implies that inter-dealer trading does not occur for \(\forall x\). Second, note that the payoff from the candidate equilibrium with \(\delta^{***}\) yields a strictly greater expected payoff to dealers than \(\delta^*\). This implies that the \(x\) at which an individual dealer is indifferent between \(\delta^{***}\) and deviating to some \(\delta_i > D - x\) is greater than \(x^{\text{trade}}\). Hence, it follows from the proof of Theorem 1 that there exists some \(x^{\text{trade, disclosure}} > x^{\text{trade}}\) such that an equilibrium with inter-dealer equilibrium exists under disclosure.

A.5 Proof of Lemma 3

Suppose that \(x > 3\Delta\) and \(D > \frac{2x^2 + \Delta^2}{\Delta}\). To ease notation, we use the following shorthand notation: \(a = \frac{D + x - \delta}{2D}, b = \frac{D - x - \delta}{2D},\) and \(c = \frac{\hat{\delta}}{D}\) and furthermore use subscripts 1 and 2 such that \(y_1 = \lambda y\) and \(y_2 = (1 - \lambda) y\) for \(y = a, b, c\). Given some market making strategy \(\delta \in [0, D - x]\) at \(t = 1\), let dealers’ interdealer strategies
be such that:

- An uninformed short dealer makes the offer \((buy, P_s^d)\), and only accepts \((sell, P)\) for \(P \leq P_s^d\).
- An uninformed long dealer makes the offer \((sell, P_l^d)\), and only accepts \((buy, P)\) for \(P \geq P_l^d\).
- An informed dealer makes the offer \((buy, P_s^d)\) if \(v = \bar{v} + x\), and otherwise makes the offer \((sell, P_l^d)\), and accepts any offer that satisfies his reservation price.
- An uninformed neutral dealer does not make or accept any offers.

Correspondingly, let dealers’ (off equilibrium) beliefs be such that:

- Offers of \((buy, P_s^d)\) are from an uninformed short dealer or an informed dealer when \(v = \bar{v} + x\). Any other buy offer (off equilibrium) is made by an informed dealer when \(v = \bar{v} + x\).
- Offers of \((sell, P_l^d)\) are from an uninformed long dealer or an informed dealer when \(v = \bar{v} - x\). Any other sell offer (off equilibrium) is made by an informed dealer when \(v = \bar{v} - x\).

Let the conjectured equilibrium prices \(P_s^d, P_l^d\) be the reservation prices of an uninformed receiving dealer of the opposite position, which are \(\bar{v} + \frac{\lambda}{2a_2 + \lambda} x - \Delta\) and \(\bar{v} - \frac{\lambda}{2a_2 + \lambda} x + \Delta\), respectively. Since \(P_s^d > \bar{v} - x + \Delta\) and \(P_l^d < \bar{v} + x - \Delta\) if \(x > 2\Delta\), any informed dealer accepts \((buy, P_s^d)\) if \(v = \bar{v} - x\) and \((sell, P_l^d)\) if \(v = \bar{v} + x\). For any uninformed dealer not of the opposite position, since their reservation price is \(P_s^d + \Delta, P_l^d - \Delta\) for buy and sell offers, respectively, they do not have an incentive to accept the offer.

We need to check that deviations by offering dealers are not profitable given beliefs. Without loss of generality, consider long dealers’ incentives to deviate. An uninformed long dealer can deviate to a screening price by offering \((sell, \bar{v} + x + \Delta)\), which would only be accepted by an informed short dealer when \(v = \bar{v} + x\).

Given \(\delta\), such a deviation is not profitable so long as

\[
\begin{align*}
  a_2 \left( \bar{v} - x - P^b - \Delta \right) + b_2 \left( a_1 [P_l^d - P^b] + (1 - a_1) [\bar{v} + x - P^b - \Delta] \right) \\
  \leq a_2 \left( b_2 [P_l^d - P^b] + (1 - b_2) [\bar{v} - x - P^b - \Delta] \right) + \\
  b_2 \left( (a_2 + \lambda) [P_l^d - P^b] + (1 - \lambda - a_2) [\bar{v} + x - P^b - \Delta] \right),
\end{align*}
\]

which requires \(\lambda \leq \frac{2a_\Delta}{x - \Delta + 3a_\Delta}\). Any \(P_l^d < P_l^d\) is dominated since the acceptance probability is unchanged. Deviating to no trade is profitable if \(a_2 \left( P_l^d - (\bar{v} - x - \Delta) \right) + \)
\((a_2 + \lambda) \left( P^d_t - (\bar{\sigma} + x - \Delta) \right) \geq 0\) which requires \(\lambda < \frac{2a\Delta}{x-\Delta+2a\Delta}\). Consider an informed long dealer when \(v = \bar{\sigma} + x\). A deviation to \((\text{sell}, P^d_t)\) is more profitable than \((\text{buy}, P^d_s)\), i.e.

\[
(a_2 + \lambda)[P^d_t - P^a_t] + (1 - a_2 - \lambda)[\bar{\sigma} + x - P^a_t - \Delta] \geq b_2[2(\bar{\sigma} + x) - P^d_s - P^a_t - 2\Delta] + (1 - b_2)[\bar{\sigma} + x - P^a_t - \Delta]
\]

which never holds since \(x > 2\Delta\). Finally, an informed short or long dealer prefers pooling prices without netting benefits yields greater conditional profits than screening prices (with netting benefits) if:

\[
a_1[P^d_t] + (1 - a_1)[\bar{\sigma} + x - \Delta] \leq (b_2[2(\bar{\sigma} + x) - 2\Delta] + (1 - b_2)[\bar{\sigma} + x - \Delta])
\]

which requires \(\lambda < \frac{bx + a\Delta - \sqrt{(a\Delta)^2 + bx\Delta}}{bx - \Delta + 2a\Delta}\) as long as \(bx - \Delta + 2a\Delta > 0\), which holds as long as \(a > \frac{x^2 + 2\Delta}{D(x + 2\Delta)}\). This requires \(\delta < \frac{Dx - x^2 + 2\Delta}{x + 2\Delta}\). Given \(a > \frac{x^2 + 2\Delta}{D(x + 2\Delta)}\), we show that for \(x > 3\Delta\) and \(D > \frac{3x^2 + \Delta^2}{\Delta}\), the uninformed dealers condition is more binding than the informed dealer condition. \(x > 3\Delta\) implies \(\frac{2a\Delta}{x-\Delta+3a\Delta} < \frac{2a}{2+3a}\). It suffices to show:

\[
\frac{bx + a\Delta - \sqrt{(a\Delta)^2 + bx\Delta}}{bx - \Delta + 2a\Delta} > \frac{2a}{2+3a}
\]

Reorganizing and simplifying the inequality:

\[
(2 - a)bx - 2a\Delta \left( \frac{2a}{2+3a} \right) > 0
\]

Since \(b = a - \frac{x}{\Delta} > \frac{\Delta(D-2x)}{D(x+2\Delta)}\) the RHS is greater than \((2 - a)\frac{\Delta(D-2x)}{D(x+2\Delta)}x - 2a\Delta \left( \frac{2a}{2+3a} \right)\).

Furthermore, since this expression decreases in \(a\) and \(a < \frac{D+x}{2D}\), it is greater than \((2 - \frac{D+x}{2D})\frac{\Delta(D-2x)}{D(x+2\Delta)}x - 2\frac{D+x}{2D} \Delta \left( \frac{2\Delta+D}{2+3\Delta} \right)\). This expression increases in \(D\), which implies that it is greater than when substituting \(D\) with \(\frac{2x^2 + \Delta^2}{\Delta}\). For any \(x > 3\Delta\), the RHS is greater than 0.

Together this implies that for \(\delta \in [0, \frac{Dx - x^2 + 2\Delta}{x + 2\Delta}]\), an equilibrium with the the proposed inter-dealer trading strategies exists for \(\lambda < \frac{2\Delta}{x-\Delta+3a\Delta}\).

### A.6 Proof of Lemma 4

We use the following shorthand notation: \(a = \frac{D+x-\delta}{2D}, b = \frac{D-x-\delta}{2D}\), and \(c = \frac{\delta}{\Delta}\) and furthermore use subscripts 1 and 2 such that \(y_1 = \lambda y\) and \(y_2 = (1 - \lambda)y\) for
\[ y = a, b, c. \] Given some market making strategy \( \delta \in [0, D - x) \) at \( t = 1 \), let dealers’ interdealer strategies be such that:

- An uninformed short dealer makes offer \((buy, \bar{v} - x - \Delta)\), and only accepts \((sell, P)\) for \( P \leq \bar{v} - x + \Delta\). An uninformed long dealer makes offer \((sell, \bar{v} + x + \Delta)\), and only accepts \((buy, P)\) for \( P \geq \bar{v} + x - \Delta\).

- An informed short dealer makes offer \((buy, v - \Delta)\), and accepts \((sell, P)\) for \( P \leq v + \Delta\). An informed long dealer makes offer \((sell, v + \Delta)\), and accepts \((buy, P)\) for \( P \geq v - \Delta\).

- A neutral dealer does not make or accept any offers.

Correspondingly, let dealers’ beliefs be such that:

- Offers of \((buy, P_s^d = \bar{v} + x - \Delta)\) are made by informed short dealers if \( v = \bar{v} + x \), and all offers \((buy, P_s^d = \bar{v} - x - \Delta)\) are made by informed short dealers if \( v = \bar{v} - x \) and uninformed short dealers. Any other buy offer (off equilibrium) is made by an informed short dealer if \( v = \bar{v} + x \).

- Offers of \((sell, P_l^d = \bar{v} - x + \Delta)\) are made by informed long dealers if \( v = \bar{v} - x \), and all offers \((sell, P_l^d = \bar{v} + x + \Delta)\) are made by informed long dealers if \( v = \bar{v} + x \) and uninformed long dealers. Any other sell offer (off equilibrium) is made by an informed long dealer if \( v = \bar{v} - x \).

Given these beliefs, consider the receiving dealer strategies. First, when an informed short (long) dealer is matched to an informed long (short) dealer, then trade always occurs, since the price is the reservation price of the receiving dealer. Also note that an informed neutral dealer does not accept any offer made by an informed dealer, since there is no surplus from trade, which is necessary to accept the additional \( \Delta \) component of the price. Since an informed neutral dealer never accepts, neither does an uninformed neutral dealer. Second, when an uninformed short (long) dealer is matched to an informed long (short) dealer, then trade occurs only when \( v = \bar{v} \pm x \), since that is the only case where price is the reservation price of the receiving informed dealer. Third, consider when the offer is made by a dealer matched to an uninformed dealer of the opposite position. Offers \((buy, \bar{v} + x - \Delta)\) and \((sell, \bar{v} - x + \Delta)\) fully reveal the type of the dealer, and hence \( v \) so the uninformed dealer accepts. For offers \((buy, \bar{v} - x - \Delta)\) and \((sell, \bar{v} + x + \Delta)\), since the uninformed dealer cannot differentiate between an offer made by a informed and uninformed dealer, the offer is accepted only if given beliefs, the offer is weakly better than his reservation price. Without loss of generality, consider an uninformed long dealer who receives offer \((sell, \bar{v} + x + \Delta)\). Since conditional on matching with an informed short dealer, the reservation price is offered, but conditional on matching with an uninformed short dealer,
the reservation price lower than the offer, the offer must be rejected for any positive probability of being matched to an uninformed dealer.

We must verify the conditions under which the conjectured inter-dealer trading strategies are incentive compatible for the (1) uninformed short dealer, (2) uninformed long dealer, (3) informed short dealer when \( v = \bar{\sigma} - \Delta \), (4) informed short dealer when \( v = \bar{\sigma} + \Delta \), (5) informed long dealer when \( v = \bar{\sigma} + \Delta \), and (6) informed long dealer when \( v = \bar{\sigma} - \Delta \).

Consider the payoff of an uninformed short dealer that deviates to some \( P_s^d \neq P_s^d \). Under the specified beliefs, a buy offer is accepted by an uninformed long dealer and by an informed long dealer when \( v = \bar{\sigma} + x \) only if \( P_s^d \geq \bar{\sigma} + x - \Delta \), and accepted by an informed long dealer when \( v = \bar{\sigma} - x \) only if \( P_s^d \geq \bar{\sigma} - x - \Delta \). It is straightforward to see that any deviation to \( P_s^d \in (\bar{\sigma} - x - \Delta, \bar{\sigma} + x - \Delta) \) is not profitable since the probability of the offer being accepted does not improve. It suffices to check when a deviation to \( P_s^d = \bar{\sigma} + x - \Delta \) is profitable, since it dominates any greater offer. Deviation is not profitable if

\[
\frac{a_2}{a_2 + b_2} \left[ b \left( P^a - P_s^d \right) + (1 - b) \left( P^a - (\bar{\sigma} + x) - \Delta \right) \right] + \frac{b_2}{a_2 + b_2} \left[ a \left( P^a - P_s^d \right) + (1 - a) \left( P^a - (\bar{\sigma} - x) - \Delta \right) \right] \leq \frac{a_2}{a_2 + b_2} [P^a - (\bar{\sigma} + x) - \Delta] + \frac{b_2}{a_2 + b_2} [a_1 (P^a - (\bar{\sigma} - x) - \Delta) + (1 - a_1) (P^a - (\bar{\sigma} - x) - \Delta)]
\]

Reorganizing the inequality yields \((2 - \lambda)\Delta \leq x\), which holds for any \( \lambda \in [0, 1] \) as long as \( x > 2\Delta \). It suffices to check whether an informed short dealer has an incentive to deviate. Since an informed short dealer when \( v = \bar{\sigma} + x \) will trade with zero probability for \( P_s^d = \bar{\sigma} - x - \Delta \), there is no incentive to deviate. For an informed short dealer when \( v = \bar{\sigma} - x \), deviating to \( P_s^d = \bar{\sigma} + x - \Delta \) is profitable if

\[
a[P^a - (\bar{\sigma} + x - \Delta)] + (1 - a)[P^a - (\bar{\sigma} - x) - \Delta] \geq a_1 [P^a - (\bar{\sigma} - x - \Delta)] + (1 - a_1) [P^a - (\bar{\sigma} - x) - \Delta]
\]

requiring \((1 - \lambda)\Delta \geq x\), which never holds.

### A.7 Proof of Theorem 4

We use the following shorthand notation: \( a = \frac{D + x - \delta}{2D}, b = \frac{D - x - \delta}{2D}, \) and \( c = \frac{\delta}{D} \) and furthermore use subscripts 1 and 2 such that \( y_1 = \lambda y \) and \( y_2 = (1 - \lambda)y \) for \( y = a, b, c \). Given Lemmas 3 and 4, we solve for the equilibrium bid-ask spreads \( \delta^* \) and \( \delta^{**} \), corresponding to each lemma. Then, we show existence of some cutoff \( \bar{\lambda} \in (0, 1) \) whereby for \( \lambda < \bar{\lambda} \), an equilibrium with \( \delta^* \) is selected, and \( \delta^{**} \) otherwise.

Given inter-dealer trading under Lemma 3, the expected payoff of a dealer
prior to information being disseminated at $t = 2$:

$$\frac{a}{a+b} \left[ b_2 \left[ P^a - \frac{P^d + P^d}{2} \right] + (1 - b_2) \left[ P^a - (\bar{\delta} + x) - \Delta \right] \right] + \frac{(1 - \lambda)b}{a+b} \left[ \lambda \left( \frac{P^d + P^d}{2} - 2(\bar{\delta} - x) - 2\Delta \right) + (1 - b_2) \left[ P^a - (\bar{\delta} - x) - \Delta \right] \right]$$

for short

$$\lambda b \left[ \frac{P^d + P^d}{2} - (\bar{\delta} + x) - \Delta \right] + \left[ (\bar{\delta} + x) - \frac{P^d + P^d}{2} - \Delta \right]$$

for neutral

$$\frac{a}{a+b} \left[ b_2 \left[ P^a + \frac{P^d + P^d}{2} - (\bar{\delta} + x) - 2\Delta \right] + (1 - b_2) \left[ P^a - (\bar{\delta} + x) - \Delta \right] \right]$$

for long

$$\frac{a}{a+b} \left[ b_2 \left[ 2(\bar{\delta} + x) - P^b - \frac{P^d + P^d}{2} - 2\Delta \right] + (1 - b_2) \left[ (\bar{\delta} + x) - P^b - \Delta \right] \right]$$

An equilibrium requires the existence of some $\delta^*$ such that dealers do not individually have an incentive to deviate to any $\delta' \neq \delta^*$. We use $a', b', c', P'^a, P'^b$ for shorthand notation, where $a' = \frac{D + x - \delta'}{2D}$, $b' = \frac{D - x - \delta'}{2D}$, $c' = \delta' D$, $P'^a = \bar{\delta} + \delta'$, and $P'^b = \bar{\delta} - \delta'$. A dealer’s expected payoff for spread $\delta'$ given all others choosing $\delta$ is

$$\frac{D - \delta'}{D} \cdot \delta' + x \left[ \frac{D + x - \delta'}{2D} (1 - b_2) + \frac{D - x - \delta'}{2D} ((1 - \lambda)(1 - \lambda - a_2) + \lambda(1 + b_2)) + \frac{x \cdot \lambda}{D} b_2 \right]$$

$$- \Delta \left[ \frac{D + x - \delta'}{2D} (1 - b_2) + \frac{D - x - \delta'}{2D} ((1 - \lambda)(1 - \lambda - a_2) + \lambda(1 + b_2)) + \frac{x \cdot \lambda}{D} b_2 \right]$$

The first order condition with respect to $\delta'$ is given by:

$$0 = \frac{D - 2\delta'}{D} + x \left[ \frac{1 - b_2}{2D} - \frac{1}{2D} \left((1 - \lambda)(1 - \lambda - a_2) + \lambda(1 + b_2)) + \frac{1}{D} \lambda b_2 \right] \right]$$

$$- \Delta \left[ \frac{1 - b_2}{2D} - \frac{1}{2D} \left((1 - \lambda)(1 - \lambda - a_2) + \lambda(1 + b_2)) + \frac{1}{D} \lambda b_2 \right] \right]$$

$$= \frac{D - x - \delta'}{D} \left[ (1 - \lambda) \left( \lambda + (1 - \lambda) \frac{x}{D} \right) \right]$$

$$+ \frac{\Delta}{2D} \left[ (1 + \lambda) - \lambda(1 - \lambda) \frac{D - x}{D} + (1 - \lambda) \frac{\delta}{D} \right]$$

Applying symmetry, and reorganizing the equation, we attain:

$$\delta^* \cdot \frac{2D^2 + (1 - \lambda)x^2 + (1 + \lambda)D \Delta + (1 - \lambda)\lambda(x - \Delta)(D - x)}{4D - (1 - \lambda)\Delta}$$
where expected profits are given by:

\[ \Pi^*(\delta^*) = \frac{D - \delta^*}{D} \delta^* - \frac{x^2}{D} - \left[ \frac{D - \delta^*}{D} - (1 - \lambda) \frac{(D - \delta^*)^2 - x^2}{2D^2} \right] \Delta \]

By Lemma 3, a necessary condition for this equilibrium is that \( \delta^* < \frac{D - x^2 + 2x\lambda}{x + 2\lambda} \) and \( \lambda < \frac{2a^*\lambda}{x - \Delta + 3a^*\lambda} \) where \( a^* = \frac{D + x - \delta^*}{2D} \). First, we establish that \( \delta^* - \frac{D - x^2 + 2x\lambda}{x + 2\lambda} < 0 \) for any \( \lambda(0,1) \). It is straightforward to show that \( \delta^* - \frac{D - x^2 + 2x\lambda}{x + 2\lambda} < 0 \) for \( x > 3\Delta \) and \( D > \frac{2x^2 + \Delta^2}{\Delta} \). Given that, substituting \( x \) with \( 3\Delta \), it suffices to show that \(-2D^2 + 6\Delta^2 (1 - \lambda)(7 - 5\lambda) + 2D\Delta(10 + (6 - 5\lambda)\lambda)\) which increases in \( D \). Since \( D > \frac{2x^2 + \Delta^2}{\Delta} \) \( \lambda \) increases in \( a \) and for any \( \lambda < \frac{2a^\Delta}{x - \Delta + 2a^\Delta} \), \( \delta^* \) increases in \( \lambda \). Hence there exists some \( \lambda \) that solves \( \frac{2\Delta}{x - \Delta + 2\Delta} \). Since \( \delta^* \in (0, D-x) \) for \( x > 2\Delta \) and \( D > \frac{2x^2 + \Delta^2}{\Delta} \), an equilibrium exists for \( \lambda \in [0, \lambda^*] \).

Given inter-dealer trading under Lemma 4, the expected payoff of a dealer prior to information being disseminated at \( t = 2 \):

\[
\begin{align*}
&\begin{cases}
\frac{b}{a^+} [a_1 \cdot [P^a - (\bar{\sigma} - x)] + (1 - a_1) \cdot [P^a - (\bar{\sigma} - x) - \Delta]] \\
+ \frac{a}{a^+} [\lambda (b \cdot [P^a - (\bar{\sigma} + x)] + (1 - b) \cdot [P^a - (\bar{\sigma} + x) - \Delta]) + (1 - \lambda) (P^a - (\bar{\sigma} + x) - \Delta)]
\end{cases} \\
&\begin{cases}
\frac{b}{a^+} [a_1 \cdot [(\bar{\sigma} + x) - P^b] + (1 - a_1) \cdot [(\bar{\sigma} + x) - P^b - \Delta]] \\
+ \frac{a}{a^+} [\lambda (b \cdot [(\bar{\sigma} - x) - P^b] + (1 - b) \cdot [(\bar{\sigma} - x) - P^b - \Delta]) + (1 - \lambda) ((\bar{\sigma} - x) - P^b - \Delta)]
\end{cases}
\end{align*}
\]

To establish existence of the equilibrium, we must determine that for some \( \delta^{**} \), dealers do not have an incentive to deviate. A dealer’s expected payoff for spread \( \delta^* \) given all others choosing \( \delta \) is

\[
\frac{D - \delta'}{2D} [P^{a'} - P^{b'}] + \frac{D - \delta'}{2D} \frac{b'}{a^+} \left[ -a_1(\bar{\sigma} - x) - (1 - a_1)[(\bar{\sigma} - x) + \Delta] + a_1(\bar{\sigma} + x) + (1 - a_1)[(\bar{\sigma} + x) - \Delta] \right] \\
+ \frac{D - \delta'}{2D} \frac{a'}{a^+} \left[ \lambda (b(\bar{\sigma} - x) + (1 - b)[(\bar{\sigma} - x) - \Delta]) + (1 - \lambda) ((\bar{\sigma} - x) - \Delta) \right] \\
+ \frac{D - \delta'}{2D} \frac{a'}{a^+} \left[ \lambda (-b(\bar{\sigma} + x) - (1 - b)[(\bar{\sigma} + x) + \Delta]) - (1 - \lambda) ((\bar{\sigma} + x) + \Delta) \right].
\]

Plugging \( P^{a'} \) and \( P^{b'} \) into the above equation yields:

\[
\frac{D - \delta'}{D} \delta' - \frac{x^2}{D} - \left( \frac{D - \delta'}{D} - \lambda (D^2 - (\delta + \delta')D - x^2 + \delta^{'2}) \right) \frac{2D^2}{\Delta}.
\]

The first order condition with respect to \( \delta' \) is \( \frac{D - 2\delta'}{D} + \frac{\Delta}{b} + \frac{\lambda (-D + \delta)}{2D^2} \Delta = 0 \). Setting
\[ \delta' = \delta, \text{ and rearranging the equation we get } \delta^{**} = \frac{2D^2 + (2 - \lambda)\Delta D}{4D - \lambda\Delta}. \] This yields an expected profit of

\[ \Pi^{**}(\delta^{**}) = \frac{D - \delta^{**}}{D} \delta^{**} - \frac{x^2}{D} - \left( \frac{D - \delta^{**}}{D} - \frac{\lambda(D^2 - 2\delta^{**}D - x^2 + \delta^{**2})}{2D^2} \right) \Delta. \]

Since \( \delta^{**} \in (0, D - x) \) for \( x > 2\Delta \) and \( D > \frac{2x^2 + \Delta^2}{\Delta} \), an equilibrium exists. Finally, let cutoff \( \bar{\lambda} = \min\{\bar{\lambda}, \bar{\lambda}\} \), where \( \bar{\lambda} \) is given by

\[ \frac{D - \delta^*(\lambda = 0)}{D} \delta^*(\lambda = 0) - \left( \frac{D - \delta^*(\lambda = 0)}{D} - \frac{(D - \delta^*(\lambda = 0))^2 - x^2}{2D^2} \right) \Delta > \]

\[ \frac{D - \delta^{**}(\lambda = 0)}{D} (\delta^{**}(\lambda = 0) - \Delta) \]

\[ \frac{D - \delta^{**}(\lambda = 1)}{D} \delta^{**}(\lambda = 1) - \left( \frac{D - \delta^{**}(\lambda = 1)}{D} - \frac{(D - \delta^{**}(\lambda = 1))^2 - x^2}{2D^2} \right) \Delta > \]

\[ \frac{D - \delta^*(\lambda = 1)}{D} (\delta^*(\lambda = 1) - \Delta) \]

This implies that \( \bar{\lambda} \in (0, 1) \).

### A.8 Proof of Theorem 5

Suppose that \( x > 3\Delta \) and \( D > \frac{3x^2 + \Delta^2}{\Delta} \). We use the following shorthand notation: \( a = \frac{D + x - \delta}{2D} \), \( b = \frac{D - x - \delta}{2D} \), and \( c = \frac{\delta}{D} \) and furthermore use subscripts 1 and 2 such that \( y_1 = \lambda y \) and \( y_2 = (1 - \lambda)y \) for \( y = a, b, c \). We claim that for any given \( \delta \), the strategic platform strictly prefers a cost schedule that induces an interior \( \lambda = \bar{\lambda}(\delta) < 1 \). Given some arbitrary \( \delta \), consider the decision of a platform to charge some cost \( c_\theta \) for type \( \theta \) for access to information in the beginning of period \( t = 2 \). First, note that for any given \( \lambda \), a platform maximizes its profits by setting \( c_\theta \) equal to the difference between being informed and uninformed for type \( \theta \). Consider the differential payoff under the two inter-dealer trading strategies outlined in Lemmas 3 and 4. Under screening prices as in Lemma 4, the differential payoff for a long or short type is given by \( \frac{a}{a + b} b \Delta \) and zero for a neutral type. Since only long and short dealers are willing to pay, the payoff is given by \( ab\Delta \).
For pooling prices as in Lemma 3, the differential payoff is given by

\[
\begin{cases}
\frac{b}{a+b}(a_2 + b_2 + \lambda)(x - \Delta) & \text{for long or short} \\
b_2(x - \Delta) & \text{for neutral}
\end{cases}
\]  

(37)

Setting each to \(c_\theta\), the payoff is given by \(\lambda b(a_2 + b_2 + c_2 + \lambda)(x - \Delta) = \lambda b(x - \Delta)\) for \(\lambda < \frac{2a\Delta}{x-\Delta+3a\Delta}\). Hence, inducing pooling prices is more profitable if \(\frac{2a\Delta}{x-\Delta+3a\Delta} \geq a \frac{\Delta}{x-\Delta}\).

Given that the platform selects prices given by Equation 37 for which \(\bar{\lambda}(\delta)\) measure of dealers choose to obtain information, we can now characterize the equilibrium \(\delta = \delta^\circ\). Since \(c_\theta\) is set equal to the value of information at \(t = 2\) for each type, a dealer’s payoff from selecting \(\delta'\) is equal to that of the expected payoff conditional on choosing to be uninformed at \(t = 2\), which is given by

\[
\frac{D - \delta'}{2D} \left[ p^{\alpha} - P_{\theta}^{\beta} - 2(1 - b_2)(x + \Delta) \right] + \frac{D - \delta'}{2D} \left[ p^{\alpha} - P_{\theta}^{\beta} + 2(1 - \lambda - a_2)(x - \Delta) \right]
\]

\[
= \frac{D - \delta'}{2D} \left[ \delta' - (1 - (1 - \lambda)b)(x + \Delta) \right] + \frac{D - \delta'}{2D} \left[ \delta' + (1 - \lambda)(1 - a)(x - \Delta) \right]
\]

\[
= \frac{D - \delta'}{2D} \left[ \delta' - (1 - (1 - \lambda)b)(x + \Delta) \right] + \frac{D - x - \delta'}{2D} \left[ \delta' + (1 - \lambda)(1 - a)(x - \Delta) \right]
\]

Taking the FOC with respect to \(\delta'\):

\[
0 = \frac{D - 2\delta'}{D} + \left(1 - (1 - \lambda)\frac{D - x - \delta'}{2D}\right)(x + \Delta) - \left(1 - \lambda\right)\left(1 - \frac{D + x - \delta'}{2D}\right)(x - \Delta)
\]

Imposing symmetry and reorganizing this yields

\[
\delta' = \frac{2D^2 + (1 - \lambda)x^2 + \Delta D + \lambda x D}{4D - (1 - \lambda)\Delta}.
\]

Note that \(\delta'\) increases in \(\lambda\) and \(\bar{\lambda}\) decreases in \(\delta\); it can be shown that there exists a unique pair \(\lambda^\circ, \delta^\circ\) such that \(\delta^\circ = \delta'(\lambda^\circ)\) and \(\lambda^\circ = \bar{\lambda}(\delta^\circ)\). Furthermore it can be verified that \(\Pi^\circ(\delta^\circ) > 0\), where \(\Pi^\circ = \frac{D - \delta}{D - \delta} - \frac{D + x - \delta}{2D} \left(1 - (1 - \lambda)\frac{D - x - \delta}{2D}\right)(x + \Delta) + \frac{D + x - \delta}{2D} (1 - \lambda) \left(\frac{D + x + \delta}{2D}\right)(x - \Delta)\).
B The Social Planner’s Problem and Free-Riding on Liquidity

In this paper, we emphasize how externalities arise due to asymmetric information and ultimately hinder market liquidity, how disclosure has the potential to resolve these issues, and finally how partial disclosure can worsen outcomes. Importantly, disclosure is a tool that indirectly affects market efficiency by affecting dealers’ incentives on liquidity provision.

Figure 5 illustrates how dealer welfare (blue line) and trader welfare (red line) is affected over an equilibrium bid-ask spread $\delta$. In this example, there exists an equilibrium without inter-dealer trade as described in Theorem 2 – dealers rationalize no inter-dealer trading, and opt to choose a wide spread $\delta^{**} > D - x$. Notably, while $\delta^{**}$ locally maximizes at the “No-Trade Eq.” point, it is strictly dominated by smaller values of $\delta$. Indeed, full disclosure makes it incentive compatible for dealers to offer a much tighter spread $\delta^{***}$ (highlighted in Theorem 3), which also restores trade in inter-dealer markets. As shown in Figure 5, this improves dealers’ welfare to the “Disclosure” point. Together, this implies that if dealers could collectively commit to $\delta^{***}$, which they cannot without disclosure, greater welfare is attained.

Does information disclosure fully resolve inefficiencies resulting from externalities? The short answer is no. We shed light on this by taking as given the market structure, matching technology, and bargaining protocol assumed in the model, and consider a social planner’s solution.

Consider a social planner who can enforce all dealers to select a bid-ask spread $\delta_i$, subject to dealers’ participation constraints. We assume that the social planner is uninformed about $v$, as are all dealers. As such, any welfare gains that arise from the social planner’s solution are necessarily drawn from the limitation of disclosure as a means of internalizing the positive externalities associated with providing liquidity, which individual dealers fail to take into account. First, we characterize the solution to the social planner’s problem where only dealer welfare is taken into consideration.

**Theorem 6 (Social Planner’s Problem: Dealer Welfare).** Suppose that the social planner maximizes dealer welfare. The social planner selects $\delta_{soc,D}^{*} = \frac{D^2}{2D - \Delta}$ for $x < \bar{x}_{soc,D}$, and $\delta_{soc,D}^{*} = x + \frac{D + \Delta}{2}$ otherwise. Furthermore, $\delta_{soc,D}^{*} < \delta^{***}$, and $\bar{x}_{soc,D}^{*} \geq x_{trade,disclosure}$.

**Proof.** We solve the social planner’s problem that maximizes ex-ante dealer welfare by characterizing the optimal $\delta$ conditional on $\delta \in (0, D - x)$ and $\delta \geq D - x$, then identifying the conditions under which each solution is the globally payoff maximizing solution.
Figure 5: Equilibrium Bid-Ask Spread and Agent Welfare. This figure provides an example where $D = 29$, $x = 7.794$, and $\Delta = 7.575$. The red and blue lines correspond to traders’ and dealers’ welfare given equilibrium bid-ask spread $\delta$, respectively. Over the blue shaded region, decreases in $\delta$ from $\delta^\text{notrade}$ to $\delta^\text{Dis}$ (i.e. $\delta^\text{trade,disclosure}$) to $\delta^\text{Soc,D}$ improve welfare of both traders and dealers. Over the red shaded region, a decrease in $\delta$ corresponds to a redistribution from dealer to trader welfare. For $\delta$ too small, corresponding to the gray shaded region, dealers’ participation condition is violated.

Given Lemma 1, it suffices to find the payoff maximizing solution conditional on the inter-dealer profit to dealers. Start with the profit equation

$$
\Pi^D(\delta) = 2 \cdot \frac{D - \delta}{2D} \left( \delta - \left( \frac{D + \delta}{2D} + \frac{x^2}{2D(D - \delta)} \right) \cdot \left[ x^2 \cdot \frac{2D}{D^2 - \delta^2 + x^2 + \Delta} \right] \right)
$$

$$
= \left[ \frac{(D - \delta)\delta}{D} - \frac{x^2}{D} - \frac{D^2 - \delta^2 + x^2}{2D^2} \Delta \right].
$$

(38)
The FOC with respect to $\delta$ yields $\delta = \frac{D^2}{2D - \Delta}$. For segmented markets:

$$\Pi^D(\delta) = 2 \cdot \frac{D + x - \delta}{4D} (\delta - x - \Delta).$$

The FOC with respect to $\delta$ yields $\delta = x + \frac{D + \Delta}{2}$.

Theorem 6 shows that the social planner chooses a tighter bid-ask spread relative to that with full disclosure, and selects a partially-revealing market making strategy for a wider interval of $x$. The social planner, by enforcing greater liquidity provision at $t = 1$, further improves expected gains from inter-dealer trade. A strict welfare enhancement arises due to the planner internalizing the benefits of greater liquidity in inter-dealer markets. Importantly, the planner’s solution highlights that disclosure cannot fully internalize the liquidity externalities. The gap between $\delta^{***}$ and $\delta^{soc,D}$ reflects the additional benefit of internalizing the cost of dealers’ private incentives to “free ride” on liquidity. If all dealers select a smaller $\delta$, the gains from trade in inter-dealer markets increase, as the mass of inter-dealer matches for which trade yields positive surplus increases. In effect, dealers individually fail to internalize the benefit of improving inter-dealer liquidity, as they prefer higher liquidity in inter-dealer markets without contributing. Even under full disclosure, where no information asymmetry exists in inter-dealer markets, dealers fail to internalize this additional benefit of collectively selecting a smaller $\delta$.

This provides scope for a social planner, who is equally uninformed about the state $v$ to eradicate welfare loss simply by requiring agents to execute a tighter bid-ask spread as in Theorem 6 and even improve on full disclosure. As shown in Figure 5, the social planner’s solution that maximizes dealer welfare, $\delta^{soc,D}$, is strictly less than that achieved through disclosure, $\delta^{***}$.

Given our main focus on the strategic interaction between dealers, we have largely abstracted from traders’ welfare. It is straightforward to see that traders unambiguously benefit from lower equilibrium $\delta$ – not only do more traders find dealers’ offers more attractive, but those who ultimately trade also extract a larger fraction of surplus from trade, as lower $\delta$ reflects better prices. This can be seen in Figure 5, where trader welfare (in red) strictly decreases in $\delta$.

The blue shaded region in Figure 5 corresponds to the interval of $\delta$ for which there is a strict improvement in all agents’ welfare as we shift from the competitive outcome, to that with full disclosure, and to the planner’s solution $\delta^{soc,D}$ that maximizes dealer welfare. In this region, which broadly corresponds to the scope of the paper, increased equilibrium market liquidity corresponds to greater welfare for all market participants. When $\delta$ drops below $\delta^{soc,D}$, a trade-off arises between market liquidity, which improves trader welfare, but lowers dealer welfare. As a starting point, consider the solution to the social planner’s problem where trader welfare maximized:
Theorem 7 (Social Planner’s Problem: Trader Welfare). The social planner selects some $\delta^{soc,T} \leq \delta^{soc,D}$ for $x < x^{soc,T}$, and $\delta^{soc,T} = x + \Delta$ otherwise, where $x^{soc,T} \geq x^{soc,D}$.

Proof. Traders’ expected payoff can be written as

$$\begin{cases} 
\frac{1}{2} (D-x-\delta)^2 + (D+x-\delta)^2 & \text{if } \delta < D-x \\
\frac{(D+x-\delta)^2}{2D} & \text{if } \delta \geq D-x.
\end{cases} \quad (40)$$

Straightforwardly, we can see that for each case, the $\delta$ value that maximizes payoff is 0 and $D-x$, respectively. Hence, it suffices to solve the social planner’s problem that maximizes trader welfare by characterizing the minimum $\delta$ that satisfies dealers’ participation conditions. Note, such $\delta$ is less than $\delta^{soc,D}$ for which dealer expected payoff is positive. Next, note that since the payoff is strictly greater when $\delta < D-x$ than $\delta > D-x$, $x^{soc,T}$ is set such that for $\delta = D-x^{soc,T}$,

$$\frac{(D-x)\delta}{D} - \frac{(x^{soc,T})^2}{D} - D^2 \cdot (x^{soc,T})^2 - 2D^2 \Delta = 0.$$

It follows directly that $x^{soc,T} > x^{soc,D}$. $\square$

When the social planner cares exclusively about trader welfare, she minimizes the bid-ask spread $\delta$, subject to dealers’ participation constraint. As a consequence, the social planner chooses an equilibrium bid-ask spread that is less than in Theorem 6, and also selects an equilibrium with inter-dealer trading for a greater interval of $x$. The difference in the solutions of Theorem 6 and Theorem 7 results from the elimination of dealer market power, which impedes market liquidity. More generally, we can explicitly express the planner’s solution to any generic reservation utility $\bar{u}$ to dealers. Given equilibrium $\delta \in [\delta^{soc,T}, \delta^{soc,D}]$, a dealer’s expected payoff is given by $\Pi(\delta) = \delta \cdot \frac{D-\delta}{D} - x \cdot \frac{x}{D} - \Delta \cdot \frac{D^2 + x^2 - \delta^2}{2D\Delta}$. Hence, for any arbitrary $\bar{u} \in [0, \Pi(\delta^{soc,D})]$, the planner’s solution $\delta^{soc}$ in $[\delta^{soc,T}, \delta^{soc,D}]$ to maximizing traders’ welfare, subject to $\bar{u}$ is given by

$$\delta^{soc}(\bar{u}) = 1 - \left[ 1 - 4 \left( \frac{1}{D} - \frac{\Delta}{2D^2} \right) \left( \frac{x^2}{D} + \frac{D^2 + x^2 - \Delta - \bar{u}}{2D\Delta} \right) \right]^\frac{1}{2}, \quad (41)$$

where $\delta^{soc}(\bar{u})$ increases in $\bar{u}$.

The above shows how for $\delta < \delta^{soc,D}$, a strict trade-off exists between trader welfare, which monotonically improves with lower $\delta$, and dealer welfare, which decrease as $\delta$ drops below the profit-maximizing equilibrium bid-ask strategy. The red shaded region in Figure 5 corresponds the interval of $\delta \in [\delta^{soc,T}, \delta^{soc,D}]$ for which a planner’s solution results in a strict trade-off between dealer and trader welfare. Reducing $\delta$ beyond $\delta^{soc,D}$ results in a redistribution of profits from dealers to traders. Whether increasing equilibrium market liquidity, by enacting policies that induce a $\delta$ lower than $\delta^{soc,D}$, is desirable depends on the Pareto weight assigned between traders and dealers.
C Co-existence equilibrium with and without inter-dealer trading

Here we show that for \( \Delta \in (\frac{D}{1+2\sqrt{2}}, D) \), \( x_{\text{notrade}} < x_{\text{trade}} \). We use the fact that \( \Delta > x_{\text{notrade}} \) and show that for any \( \Delta > x_{\text{notrade}} \) the condition is satisfied. For \( \delta(\delta^*) = x + \frac{D+\Delta - (x+\Delta)(D-x-\delta^*)}{2D} \), the condition is given by

\[
\frac{(D-\delta^*)(\delta^*-\Delta)}{D} - \frac{x^2}{D} + \frac{(D+x-\delta^*)(D-x-\delta^*)}{2D^2} \Delta > \frac{(D+x-\delta)(\delta-x-\Delta)}{2D} + \frac{(D-x-\delta^*)(x-\Delta)}{4D^2} (x+\Delta).
\]

Since \( \Delta > x_{\text{notrade}} \),

\[
\frac{(\delta-\delta^*)(D-x-\delta^*)}{2D^2} \Delta > \frac{(D+x-\delta)(\delta-x-\Delta)}{2D} - \frac{(D-\delta^*)(\delta^*-\Delta)}{D} + \frac{x^2}{D}.
\]

Substituting in \( \delta(\delta^*) \),

\[
\frac{(\delta-\delta^*)(D-x-\delta^*)}{2D^2} \Delta > \left( \frac{D-\Delta}{2} \right)^2 + \frac{(D-x-\delta^*)(x+\Delta)}{2D} (-D-x+\delta-x-\Delta) - \frac{(D-\delta^*)(\delta^*-\Delta)}{D} + \frac{x^2}{D}.
\]

Reorganizing the inequality,

\[
\frac{(D-\delta^*)(\delta^*-\Delta)}{D} - \frac{x^2}{D} + \frac{(D+2x+\Delta-\delta^*)(D-x-\delta^*)}{2D^2} \Delta > \frac{x^2}{D} - \frac{(D-x-\delta^*)(x+\Delta)}{2D^2}.
\]

Note, since the RHS is less than payoff conditional on deviating to \( \delta = \frac{D+\Delta}{2} \) without inter-dealer trading, and the LHS is the payoff conditional on \( \delta^* \), the inequality strictly holds.

This implies that an equilibrium with inter-dealer trading always exists at threshold \( x_{\text{notrade}} \). Hence, for \( \Delta > \frac{D}{1+2\sqrt{2}} \), there exists a nonempty interval of \( x \) for which both types of equilibria exist.