14.04 Intermediate Micro Theory: Lecture 12

Prediction: Walrasian Equilbrium and International Trade

Robert M. Townsend

MIT

Fall 2019

Agenda

Today:

- Define Walrasian Equilibrium
- Application: Production in Small Open Economy

General Setup

- Let $L > 0$ be the finite number of commodities in an economy
- Let $I > 0$ be the number of consumers
- Let $J \geq 0$ be the number of firms
- Each consumer $i = 1, 2, ..., I$ has a consumption set $X_i \subseteq \mathbb{R}_+^L$ and preferences $\succeq_i$ over bundles on X_i
- Each firm $j = 1, 2, ..., J$ has a technology, characterized by a production set $Y_i \subseteq \mathbb{R}^L$
- The economy resources are given by a vector of aggregate endowments $\overline{\omega} = (\overline{\omega}_1, \overline{\omega}_2, ..., \overline{\omega}_L) \in \mathbb{R}^L$

Private Ownership Economies

- In the private ownership economy, each consumer i has an endowment vector $\omega_i \subseteq \mathbb{R}_+^L$ such that $\sum_{i=1}^{I} \omega_i = \overline{\omega}$
- Each consumer i also has an ownership share θ_{ij} of firm j''s profit. That is, if profits of firm j are π_j, then consumer i gets $\theta_{ij}\pi_j$ of its profits. We must have $\sum_{i=1} \theta_{ij} = 1$, so that all of each firm is owned by someone.
- So, a private ownership economy can be summarized by

$$\mathscr{E} \equiv \left\{ \{X_i, \succeq_i, \omega_i\}_{i=1}^{I}, \{Y_j\}_{j=1}^{J}, \{\theta_{ij}\}_{i=1,j=1}^{I,J} \right\}$$

Walrasian Equilibrium

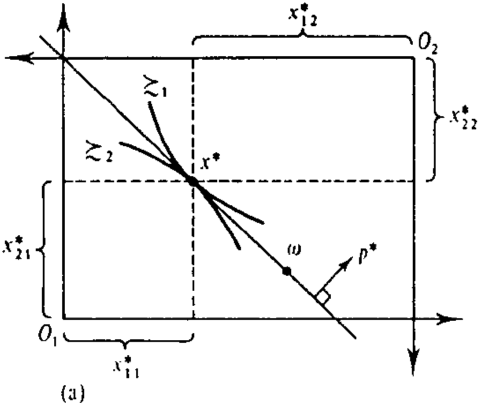
Let $p \in \mathbb{R}^L_+$ be the price vector of all commodities. We define a Walrasian equilibrium for the private ownership economy $\mathscr{E}$ as follows:

Definition

A Walrasian equilibrium is an allocation (x^*, y^*) and a price vector $p^* \in \mathbb{R}^L_+$ such that:

1. For each j, y_j^* maximizes profits given prices p^*: $p^* \cdot y_j^* \geq p^* \cdot y_j$ for all $y_j \in Y_j$
2. For each i, x_i^* maximizes preferences given her budget: $x_i^* \succeq_i x_i$ for all $x_i \in X_i$ such that $p^* \cdot x_i \leq p^* \cdot \omega_i + \sum_j \theta_{ij} p^* \cdot y_j^*$
3. The allocation is feasible: $\sum_i x_i \leq \sum_i \omega_i + \sum_j y_j^*$

Equilibrium in Edgeworth Box



Here we represent the initial endowment ω, the equilibrium price vector p^* and the equilibrium allocation x^* for a Walrasian equilibrium in a pure exchange economy with 2 consumers.

Price Equilibrium with Transfers

Definition

An allocation (x^*, y^*) and a price vector $p \in \mathbb{R}^L_+$ constitute a price equilibrium with transfers if there exists an *assignment* of wealth levels $w = (w_1, w_2, ..., w_I)$ such that

1. For each j, y_j^* maximizes profits given prices. (That is, $p \cdot y_j^* \geq p \cdot y_j$ for all $y_j \in Y_j$.)
2. For each i, x_i^* maximizes preferences given her budget. (That is, $x_i^* \succeq_i x_i$ for all $x_i \in X_i$ such that $px_i \leq w_i$.)
3. The allocation is feasible. (That is, $\sum_i x_i \leq \bar{\omega} + \sum_j y_j^*$)
4. The assignment of wealth levels is feasible: $\sum_i w_i = p \cdot \bar{\omega} + \sum_j p \cdot y_j^*$.

Note that any Walrasian equilibrium is a price equilibrium with transfers, where $w_i = p \cdot \omega_i + \sum_j \theta_{ij} p \cdot y_j^*$.

Statement of the Welfare Theorems

1. All Walrasian equilibria are Pareto Optimal
2. Any Pareto Optimum supported as a price equilibrium with transfers

Application: Production in Small Open Economy

- Consider an economy with J firms
- Each firm j produces a consumer good q_j from a vector of L factors, $z_j = (z_{1j}, ..., z_{Lj}) \geq 0$. Each firm j has a concave, strictly increasing, and differentiable production function $f_j(z_j)$
- The economy has total endowments of the L factor inputs, $\bar{z} = (\bar{z}_1, ..., \bar{z}_L) \gg 0$. These endowments are initially owned by consumers and cannot be consumed directly
- The prices of the J produced consumption goods are fixed at $p = (p_1, ..., p_J) \gg 0$
- Think: small open economies whose trading decisions in the world markets for consumption goods have little effect on the world prices of these goods. Output is sold in world markets. Factors are immobile (i.e. traded locally) and must be used for production within the country
- Goal: Compute Walrasian equilibrium factor prices w^* and allocation z^*

Walrasian Equilibrium Conditions

- An equilibrium for the factor markets of this economy given the fixed output prices p consists of an input price vector w^* and a factor allocation z^* such that firms receive their desired factor demands under prices (p, w^*) and all the factor markets clear.
- That is, for all j,

$$z_j^* \in \arg\max_{z_j} p_j f_j\left(z_j\right) - w^* \cdot z_j$$

  and for all l,

$$\sum_j z_{lj}^* = \bar{z}_l$$

First Order Conditions

- Because of the concavity of firms' production functions, first-order conditions are both necessary and sufficient for the characterization of optimal factor demands.
- Therefore, the $L(J+1)$ variables formed by the factor allocation z^* and the factor prices w^* constitute an equilibrium if and only if they satisfy the following $L(J+1)$ equations (we assume an interior solution here): That is, for all firms and factors, i.e. for all j, l,

$$p_j \frac{\partial f_j\left(z_j^*\right)}{\partial z_{lj}} = w_l^*$$

and for all factors l,

$$\sum_j z_{lj}^* = \bar{z}_l.$$

The equilibrium output levels are then $q_j^* = f_j\left(z_j^*\right)$ for every j.

Cost Function Formulation

- Recall the cost minimization framework from lecture 4
- Equilibrium conditions for outputs and factor prices can alternatively be stated using the firms' cost functions $c_j\left(w, q_j\right)$ for $j = 1, ..., J$.
- That is, for all firms j,

$$p_j = \frac{\partial c_j\left(w^*, q_j^*\right)}{\partial q_j}$$

and for all factors l,

$$\sum_j \frac{\partial c_j\left(w^*, q_j^*\right)}{\partial w_l} = \bar{z}_l.$$

- The second equation comes from Shephard's lemma

Shephard's Lemma

Theorem (Shephard's Lemma)

Suppose that $c(w, y)$ is continuously differentiable in w (for fixed y) at price vector w^*. Let x^* be any solution of the cost minimization problem. Then

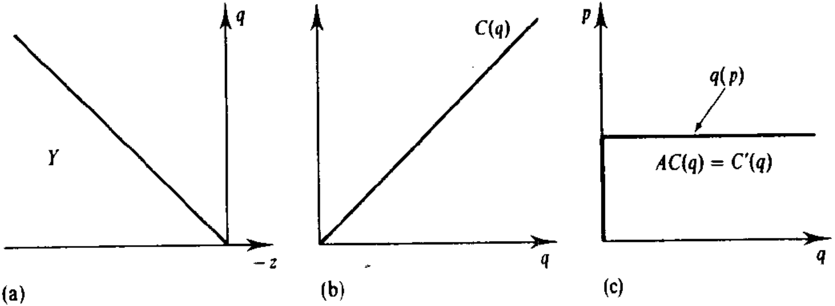
$$\frac{\partial c(w, y)}{\partial w} = x_n^*.$$

- This is essentially the same result as Hotelling's Lemma before

Special Case: 2×2 Production Model

- Let $J = L = 2$, i.e. two outputs are produced from two factors
- Factor 1 is often thought of as labor and factor 2 as capital
- We assume that the production functions $f_1(z_{11}, z_{21}), f_2(z_{12}, z_{22})$ are homogeneous of degree one (so the technologies exhibit constant returns to scale)
- For every vector of factor prices $w = (w_1, w_2)$, we define
  - $c_j(w)$ the minimum cost of producing one unit of good j
  - $a_j(w) = (a_{1j}(w), a_{2j}(w))$ the input combination (assumed unique) at which this minimum cost is reached.

Constant returns to scale (CRS)



(a) CRS production set
(b) CRS cost function $C(q)$
(c) CRS marginal cost, average cost, and supply curves

Implication of CRS: Zero profit

Under CRS production, firm profits must be zero if firm is producing.

- Proof is by contradiction: If profits are positive, then firms will expand like crazy. If profits are negative, then firms are not producing.
- This implies that
$$c_j(w) = p_j,$$
where $c_j(w)$ is the minimum cost of producing one unit of good j .

Implication of CRS: Constant factor ratios

Under CRS production, minimizing cost gives us same factor ratios regardless of the output level.

- Proof: Let the cost-minimizing input requirement for any output level be

$$z_j^*\left(w, y_j\right) = \arg\min_{z_j} w \cdot z_j \text{ s.t. } f\left(z_j\right) = y_j.$$

Note that $z_j^*\left(w, \lambda y_j\right) = \lambda z_j^*\left(w, y_j\right)$ for $\lambda > 0$ since CRS implies $f\left(\lambda z_j\right) = \lambda f\left(z_j\right)$. Recall that $a_j\left(w\right) = z_j^*\left(w, 1\right)$. This implies that

$$\frac{z_{1j}^*}{z_{2j}^*} = \frac{a_{1j}\left(w\right)}{a_{2j}\left(w\right)}.$$

Factor Intensity

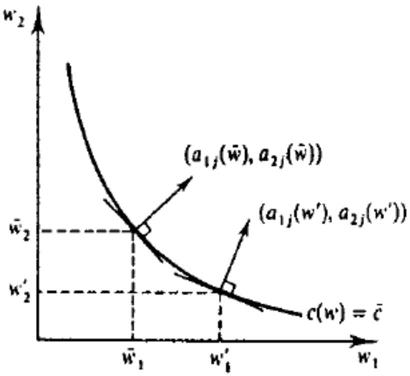
Definition

The production of good 1 is relatively more intensive in factor 1 than is the production of good 2 if

$$\frac{a_{11}(w)}{a_{21}(w)} > \frac{a_{12}(w)}{a_{22}(w)}$$

at all factor prices $w = (w_1, w_2)$.

Level Curve of the Unit Cost Function



Here we draw a level curve of the unit cost function: $\left\{(w_1, w_2) : c_j(w_1, w_2) = \overline{c}\right\}$, where $c_j(w_1, w_2)$ the minimum cost of producing one unit of good j. This curve is downward sloping because as w_1 increases, w_2 must fall in order to keep the minimized costs of producing one unit of good j unchanged. Moreover, the set $\left\{(w_1, w_2) : c_j(w_1, w_2) \geq \overline{c}\right\}$ is convex because of the concavity of the cost function $c_j(w)$ in w. (See next slide.) The vector $\nabla c_j(w)$ captures how production costs respond to a change in factor prices and is therefore perpendicular to the level curve at $\overline{w} = (\overline{w}_1, \overline{w}_2)$. Note that if we totally differentiate the cost function, we have

$$dc_j = \frac{\partial c_j}{\partial w_1} dw_1 + \frac{\partial c_j}{\partial w_2} dw_2.$$

Shephard's Lemma implies that $z^*_{1j}(w,1) = \frac{\partial c_j}{\partial w_1}$ and $z^*_{2j}(w,1) = \frac{\partial c_j}{\partial w_2}$. By definition, $a_j(w) = z^*_j(w,1)$. It follows that that $\nabla c_j(w)$ is exactly $\left(a_{1j}(w), a_{2j}(w)\right)$. As we move along the curve toward higher w_1 and lower w_2, the ratio $a_{1j}(w)/a_{2j}(w)$ falls.

Properties of the Cost Function

Theorem (Properties of the cost function)

1. *The cost function c is homogeneous of degree one in w (i.e if all factor prices double, then $c(w,y)$ doubles);*
2. *The cost function is continuous in w;*
3. *The cost function is concave in w.*

(Concavity of cost function).

Take any two prices w and w', and any scalar $\alpha \in [0,1]$, and let $w'' = \alpha w + (1-\alpha) w'$. Let x solve $CMP(w'', y)$, so that $c(w'', y) = w'' \cdot x$. Since x remains feasible at prices w and w', $c(w,y) \le w \cdot x$ and $c(w', y) \le w' \cdot x$. Therefore

$$c\left(w'', y\right) = w'' \cdot x = \left(\alpha w + (1-\alpha) w'\right) \cdot x \ge \alpha c(w,y) + (1-\alpha) c\left(w', y\right).$$

□

Solving for Equilibrium Factor Prices

- To determine the equilibrium factor prices, suppose that we have an interior equilibrium in which the production levels of the two goods are strictly positive
- Given our constant returns assumption, a necessary condition for (w_1^*, w_2^*) to be the factor prices in an interior equilibrium is that it satisfies the system of equations

$$c_1(w_1, w_2) = p_1 \text{ and } c_2(w_1, w_2) = p_2$$

- Interpretation: at unity production revenue is price p and must be equal to cost
- This gives us two equations for the two unknown factor prices w_1 and w_2

Solving for Equilibrium Factor Prices

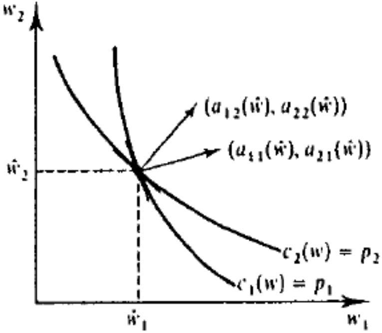


Figure depicts the level curves for the two unit cost functions. A necessary condition for $(\hat{w}_1, \hat{w}_2)$ to be the factor prices of an interior equilibrium is that these curves cross at $(\hat{w}_1, \hat{w}_2)$. Moreover, the factor intensity assumption implies that whenever the two curves cross, the curve for firm 2 must be flatter (less negatively sloped) than that for firm 1. (Proof by contradiction using factor intensity definition.) From this. it follows that the two curves can cross at most once.

Unit isoquant

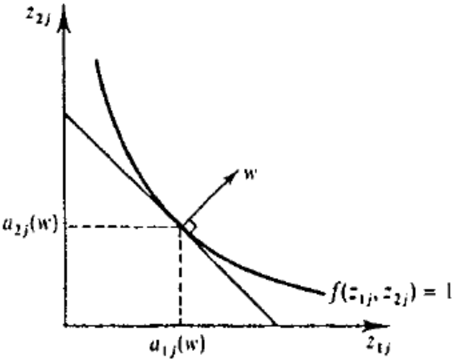
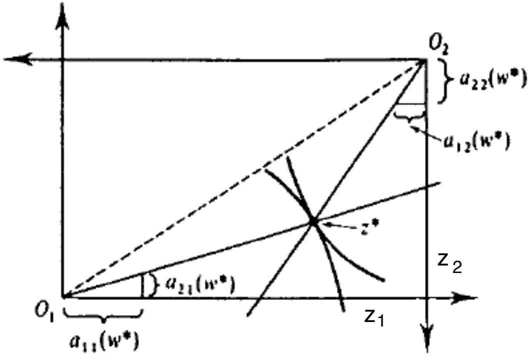


Figure depicts the unit isoquant of firm j, $\left\{\left(z_{1j}, z_{2j}\right) \in \mathbb{R}_{+}^{2} : f_j\left(z_{1j}, z_{2j}\right) = 1\right\}$, along with the cost-minimizing input combination $\left(a_{1j}(w), a_{2j}(w)\right)$

Solving for Equilibrium Factor Allocations

- Once equilibrium factor prices w^* are known, the equilibrium output levels can be found by determining the unique point (z_1^*, z_2^*) such that
$$\frac{z_{11}^*}{z_{21}^*} = \frac{a_{11}(w^*)}{a_{21}(w^*)} \text{ and } \frac{z_{12}^*}{z_{22}^*} = \frac{a_{12}(w^*)}{a_{22}(w^*)}$$
and
$$z_{11}^* + z_{12}^* = \overline{z}_1 \text{ and } z_{21}^* + z_{22}^* = \overline{z}_2.$$
- The first condition comes from cost minimization FOC under CRS production. The second is due to market clearing.

Solving for Equilibrium Factor Allocations



The equilibrium factor allocation z^* in the Edgeworth box is determined by cost minimization by both firms

$$\frac{z_{11}^*}{z_{21}^*} = \frac{a_{11}(w^*)}{a_{21}(w^*)} \text{ and } \frac{z_{12}^*}{z_{22}^*} = \frac{a_{12}(w^*)}{a_{22}(w^*)}$$

combined with market clearing

$$z_{11}^* + z_{12}^* = \bar{z}_1 \text{ and } z_{21}^* + z_{22}^* = \bar{z}_2.$$

The $2 \times 2 \times 2$ model

- Two countries A and B
- Two outputs are produced from two factors
- Identical technologies and preferences in two countries
- Different factor endowments: One country is capital-abundant, the other is labor-abundant

Trade or no trade

- Autarky – The prices p^c of the two consumed goods are determined by **domestic** product market clearing. No movement of goods or factors across borders.
- Free trade – The prices p of the two consumed goods are determined by **international** product market clearing. Movements of goods across borders but no movement of factors.

Heckscher-Ohlin Theorem

Theorem (Heckscher-Ohlin Theorem)

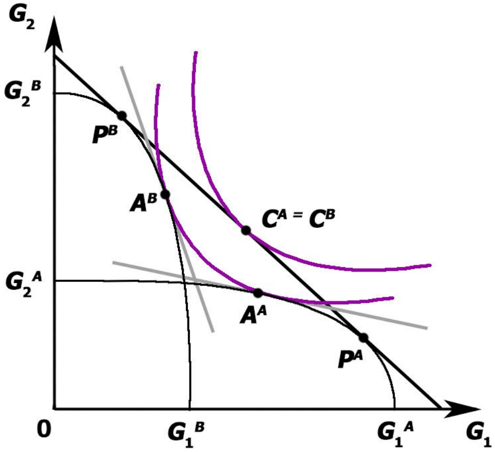
Initially, when the countries are not trading:

- *the price of the capital-intensive good in the capital-abundant country will be bid down relative to the price of the good in the other country,*
- *the price of the labor-intensive good in the labor-abundant country will be bid down relative to the price of the good in the other country.*

Once trade is allowed, profit-seeking firms will move their products to the markets that have (temporary) higher price. As a result:

- *the capital-abundant country will export the capital-intensive good,*
- *the labor-abundant country will export the labor-intensive good.*

Heckscher-Ohlin Theorem

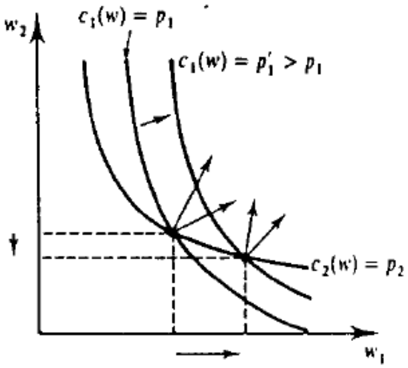


Stopler-Samuelson Theorem

Theorem (Stopler-Samuelson Theorem)

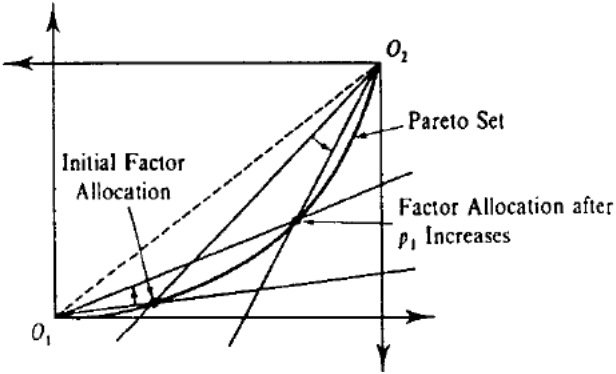
In the 2 x 2 production model with the factor intensity assumption, if p_i increases, then the equilibrium price of the factor more intensively used in the production of good i increases, while the price of the other factor decreases (assuming interior equilibria both before and after the price change).

Stopler-Samuelson Theorem



How does a change in the price of one of the outputs, say p_1, affect the equilibrium factor prices and factor allocations? The increase in p_1 shifts firm I's curve outward toward higher factor price levels; the point of intersection of the two curves moves out along firm 2's curve to a higher level of w_1 and a lower level of w_2.

Stopler-Samuelson Theorem



This figure depicts the resulting change in the equilibrium allocation of factors from an increase in p_1. As can be seen, the factor allocation moves to a new point in the Pareto set at which the output of good I has risen and that of good 2 has fallen.

Factor Price Equilization Theorem

- In the 2 x 2 production model, if the factor intensity condition holds, then as long as the economy does not specialize in the production of a single good, the equilibrium factor prices w^* depend *only on the technologies of the two firms and on the output prices p*

$$c_1(w_1^*, w_2^*) = p_1 \text{ and } c_2(w_1^*, w_2^*) = p_2$$

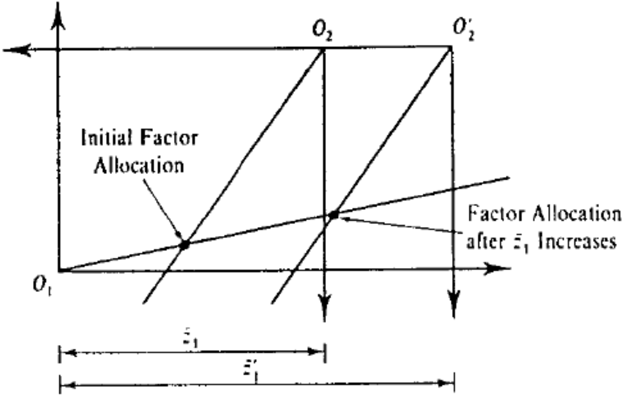
- Thus, the levels of the endowments matter only to the extent that they determine whether the economy specializes.
- This result is known in the international trade literature as the *factor price equalization theorem*
- The theorem provides conditions (which include the presence of tradable consumption goods, identical production technologies in each country, and price-taking behavior) under which the prices of nontradable factors are equalized across nonspecialized countries.

Rybcszynski Theorem

Theorem

In the 2 x 2 production model with the factor intensity assumption, if the endowment of a factor increases, then the production of the good that uses this factor relatively more intensively increases and the production of the other good decreases (assuming interior equilibria both before and after the change of endowment).

Rybcszynski Theorem



What happens when the total availability of factor 1 increases from z_1 to z_1'? Because neither the output prices nor the technologies have changed, the factor input prices remain unaltered (as long as the economy does not specialize). As a result, factor intensities also do not change. The new input allocation is then easily determined in the superimposed Edgeworth boxes; we merely find the new intersection of the two rays associated with the unaltered factor intensity levels.