Welfare Theorems Extended

14.04 Intermediate Micro Theory: Lecture 16

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Outline

Applications of Welfare Theorems in Hyperspace:

- Incentive constrained contracts
- The space of lotteries
- Welfare theorems extensions and qualifications

CD Withdrawal Options

Calculating effective return on a CD after paying early withdrawa

CD Term	2-Year	3-Year	4-Year	5-Year	7-Year	
APY	1.25%	2.25%	2.76%	3.00%	3.51%	
Withdraw after year	Effective Return					
1	0.63%	1.12%	1.38%	1.50%	0.00%	
2	1.24%	1.69%	2.07%	2.25%	1.74%	
3		2.25%	2.30%	2.50%	2.33%	
4			2.76%	2.62%	2.62%	
5				3.00%	2.80%	
6					2.92%	
7					3.51%	

Computations use simple interest (compounded annually).

Early withdrawal penalty is previous 6 months interest for 2,3,4,and 5 year CDs and last year's interest for 7-Year CD

Pension Options

Type	Description	Details
Single Life Annuity	An annuity that pays you for your lifetime until you pass away.	Typically provides highest annuity benefit, but does not provide for any survivor benefits for a spouse.
Joint and Survivor Annuity	An annuity that pays you for your lifetime until you pass away. Payments will then continue for the life of your spouse.	This will provide lower initial monthly payments than a single life annuity because payments will continue to your surviving spouse. There are also typically survivor payout options (as a % of initial benefit amount) such as 100%, 75%, or 50%. Each option will lower the spousal annuity payment accordingly. A lower spousal survivor benefit will increase the initial annuity payment amount.
Period Certain Payment	An annuity payment that is guaranteed for a specific number of years, even if you pass away. This option is not common in most corporate pension plans.	For example, if you choose a 10-year period certain, you will receive payments for 10 years, and if you die during that period, your beneficiary will receive the balance of payments.
Lump Sum Distribution	Commonly determined from a formula using interest rate and life expectancy assumptions.	Lump sum distribution can (and should) be made in a tax-free rollover into an IRA account.

General Competitive Analysis in an Economy with Private Information by Prescott and Townsend (IER 84)

- This paper extends the theory of general equilibrium in pure exchange economies to a prototype class of environments with **private information.**

and

- examines again the role of securities in the optimal allocation of risk-bearing.
- The first welfare theorem holds in this economy:
  - competitive equilibrium allocations are Pareto optimal.
- The second fundamental welfare theorem however does not hold:
  - Not all Pareto optimal allocations can be supported as competitive equilibria.

Motivating Example

- at $T = 0$ all agents are the same
- at $T = 1$ fraction $\lambda(\theta)$ of agents receive a private shock $\theta \in \Theta = \theta_1, \theta_2$
- and their utility from consumption becomes $U(c, \theta)$
  - $U(c, \theta)$ is increasing, concave and continuously differentiable in c
  - $U'(\infty, \theta_1) = 0$ and $U(c, \theta_2) = \theta_2 c$, $(\theta_2 > 0)$
    - We only require type1 to be more risk averse than type2
- all agents recieve endowment e of consumption good with certainty and $U'(e, \theta_1) < \theta_2$.

Pareto Optimal Allocation

If θ were public, Pareto optimal allocation problem at $T = 0$ is:

$$\max_{c_1, c_2} \quad \lambda(\theta_1)U(c_1, \theta_1) + \lambda(\theta_2) \times (\theta_2 c_2)$$

$$s.t.\ \lambda(\theta_1)c_1 + \lambda(\theta_2)c_2 \leq e$$

- Pareto optimal allocation requires:

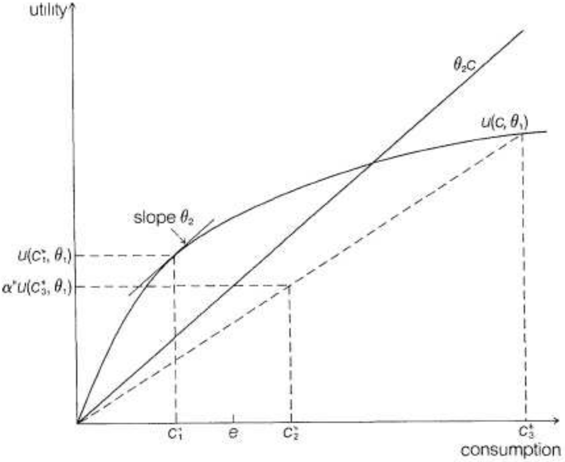
$$U'(c_1^*, \theta_1) = \theta_2$$

$$\lambda(\theta_1)c_1^* + \lambda(\theta_2)c_2^* = e$$

  - i.e. marginal utilities are equated across states and the endowment is exhausted

- But with our assumptions, this requires $c_1^* < c_2^*$.
- If θ is private knowledge, we cannot implement this allocation since type1 is always better off reporting her type is 2.

Graphical Illustration



Pareto Optimal Allocation with Lotteries

- Lotteries can solve this incentive compatibility problem.
- Since type2 is risk neutral, she is indifferent between:
  - recieving c_2^* with certainty
  - recieving c_3^* with probability $\alpha^* = c_2^*/c_3^*$ and consumption 0 with probability $1 - \alpha^*$.
- But for c_3^* sufficiently large, type1 agents prefer consuming c_1^* for sure instead of type2 agents allocation.
- Thus with lotteries we can achieve an allocation that is both Pareto optimal and incentive compatible.

Lotteries

- One can mimic the effects of a lottery by indexing on the basis of a naturally occurring random variable that is unrelated to preferences and technology, provided that the random variable has a continuous density.
- Agents are required to surrender their endowment e to the broker and then, subsequent to the revelation of the shocks, they have a choice between two distribution centers.
- If they choose the first, they are guaranteed c_1^* units of the good.
- If they choose the second, they receive c_3^* units if it is avaialble.
  - Households choosing the second center are imagined to arrive in a random fashion and to receive c_3^* on a first-come, first-served basis.
- Agents are not permitted to recontract contingent upon whether or not they are served.

Competitive Market Implementation

- Imagine households can buy and sell contracts (make commitments) in a planning period ($T = 0$) market.
- Commitments can be conditional on households' individual circumstances (i.e. their private shocks θ)
  - of course, households will choose the option which is best given its individual circumstance.
  - W.L.O.G. we can restrict to options such that household announce its individual shocks truthfully.
- We allow options to affect random allocation of consumption good.

Contracts as a Bundle-I

Without Lotteries. Simplest notation is one good, but here to make sense need vector, otherwise no trade without lotteries.

- $c(\theta)$ is the contract contingent on θ, $[c(\theta), \theta]$
- Then $U[c(\theta), \theta] \geq U[c(\theta'), \theta]$ for all $\theta, \theta' \in \Theta$
- The expected utility of contract $[c(\theta), \theta]$ for $\theta \in \Theta$ is:

$$W\{[c(\theta), \theta]\} = \sum\nolimits_{\theta} \lambda(\theta) U[c(\theta), \theta]$$

- Competitive Market
  - Households maximize in the standard problem by purchasing incentive compatible contracts $[c(\theta), \theta]\theta \in \mathit{\Theta}$, taking some pricing function $p(\theta)\theta \in \mathit{\Theta}$ as given:

$$\max \sum\nolimits_{\theta} \lambda(\theta) U[c(\theta), \theta]$$

$$\text{s.t.} \sum\nolimits_{\theta} p(\theta) c(\theta) \leq \sum\nolimits_{\theta} p(\theta) \varsigma$$

    - So it is as if selling endowment (ς) and buying θ contingent consumption back
    - Equivalent with excess demand, or supply, for each θ, hence insurance

Contracts as a Bundle-II

- A broker dealer offering contracts $[y(\theta), \theta \in \Theta]$, where $y(\theta) > 0$: broker dealer is giving out to those who announce ϑ, indemnity, ex-post
- $y(\theta) < 0$: broker dealer is taking in from those who announce θ, premium, ex-post
  - Revenue is$\sum_\theta p(\theta)y(\theta)$
  - Feasible trading set is defined by $\sum_\theta \lambda(\theta)y(\theta) \leq 0$

Competitive Market

Formally an insurance contract can be shown by

$$x(c, \theta), c \in C, \theta \in \Theta$$

- If household announce its shock θ, in the consumption period receive c with probability $x(c, \theta)$.
    - of course $0 \leq x(c, \theta) \leq 1$ and $\sum_c x(c, \theta) = 1$
- Households buy these insurance contracts in the planning period market.
- Households endowments can be shown by probability measures $\zeta(c, \theta)$, $\theta \in \Theta$ each putting mass one on the endowment point e.
- These endowments are sold in the planning period market.

Competitive Market

- In summary households maximize:

$$\max_{x(c,\theta)} \sum_{\theta} \lambda(\theta) \sum_{c} x(c,\theta) U(c,\theta)$$

$$s.t. \quad \sum_{\theta} \sum_{c} p(c,\theta) x(c,\theta) \leq \sum_{\theta} \sum_{c} p(c,\theta) \zeta(c,\theta)$$

and *incentive compatibility*

- We also assume there are firms or intermediaries that make commitments to buy and sell the consumption good.

Competitive Market

- Firm production $y(c, \theta)$ delivers c units of consumption if agent announce her type is θ.
- Production set of each firm is defined by

$$Y = \left\{ y(c, \theta), c \in C, \theta \in \Theta : \sum_{\theta} \lambda(\theta) \sum_{c} c y(c, \theta) \leq 0 \right\}$$

- (intermediary effectively facing aggregate resource constraint)
- This requires each firm not deliver more of the single consumption good in the consumption period than it takes in.
- Y displays constant return to scale. So we can assume we only have one price taker firm.
- $y(c, \theta)$ is passive:
  - $y(c, \theta) > 0$: firm is giving away.
  - $y(c, \theta) < 0$: firm is taking in.

Competitive Market

- Firm problem is:

$$\max \sum_{\theta} \sum_{c} p(c,\theta) y(c,\theta)$$

- Equilibrium price system $p^*(c,\theta)$ must satisfy

$$p^*(c,\theta) = \lambda(\theta)c$$

- This corresponds to actuarially fare insurance.
  - Price of A-D security which pays c at state θ is just equal probability of the state $\times$ consumption in that state

Welfare Theorems-I

- An allocation (x_i) is implementable if it satisfies the resource constraints and a no-envy constraint

$$W(x_i, i) \geq W(x_j, i) \; \forall i, j$$

- An allocation is a Pareto optimum if it is implementable and there does not exist an implementable allocation (x_i') such that $W(x_i', i) \geq W(x_i, i)$ with a strict inequality for some i.
- Definition of Competitive Equilibrium
  - A competitive equilibrium is
    - a state$[(x_i^*), y^*]$
    - a price system $v*$
  - such that:
    1. for every i, x_i^*maximizes $W(x_i, i)$ subject to $x_i \in X$ and $v^*(x_i) \leq v^*(\zeta)$
    2. $y*$ maximizes $v*(y)$ subject to $y \in Y$
    3. $\sum_{i=1}^{n} \lambda(i) x_i^* - y^* = \zeta$

Welfare Theorems-II

- First Welfare Theorem
  - If the allocation $[(x_i^*), y^*]$, together with the price system v^*,is a competitive equilibrium and if no $x_i^*$is a local saturation point, then $[(x_i^*), y^*]$ is a Pareto optimum.
- Second Welfare Theorem
  - With private information, there is no guarantee that every Pareto optimum can be supported by a quasi-competitive equilibrium with an appropriate redistribution of wealth.
  - It is true that a separating hyperplane exists such that y^* maximizes value subject to the technology constraint, but x_i^* does not necessarily minimize value over the set $\{x_i \in X_i : \; W(x_i, i) \geq W(x_i^*, i)\}$. Rather, it minimizes value over the set $\left\{x_i \in X_i : \; W(x_i, i) \geq W(x_i^*, i) \text{ and } W(x_i, j) \leq W(x_j^*, j) \text{ for } j \neq i\right\}$.
  - Need no envy condition.